United States
Burning Plasma Organization

# Toroidal Alfvén Eigenmode
## Existence and Implications


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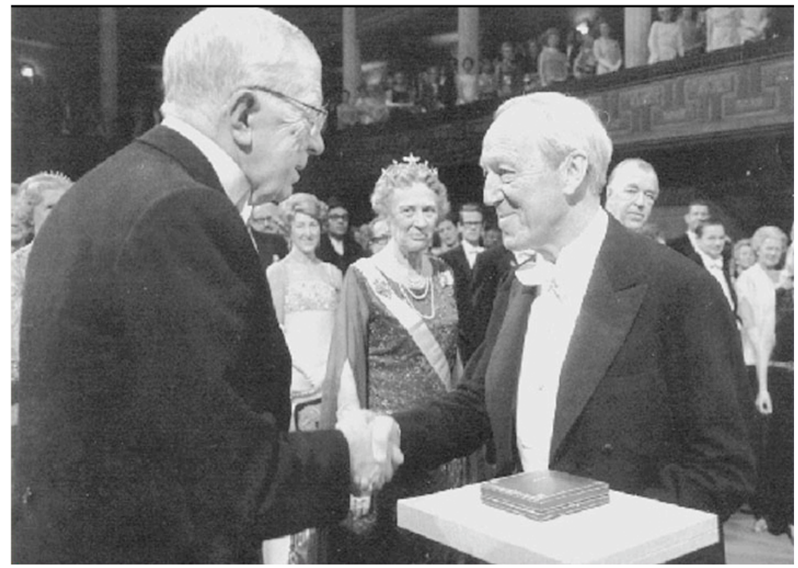
# *INTRODUCTION:*
# *Hannes Alfvén*

# "Father of Plasma Physics"

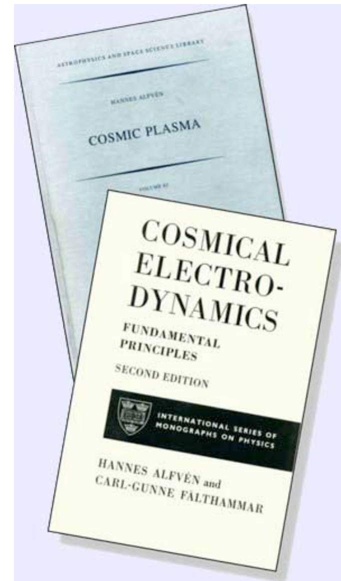
- **Hannes Olof Gösta Alfvén**
  - Born 30 May 1908 (Norrköping, Sweden); died 2 April 1995
- **Career at a glance**
  - Professor of electromagnetic theory at Royal Institute of Technology, Stockholm (194)
  - Professor of electrical engineering at University of California, San Diego (1967-1973/1988)
  - Nobel Prize (1970) for MHD work and contributions in founding plasma physics



Hannes Alfvén received the Nobel Prize in Physics in 1970 from King Gustavus Adolphus VI of Sweden

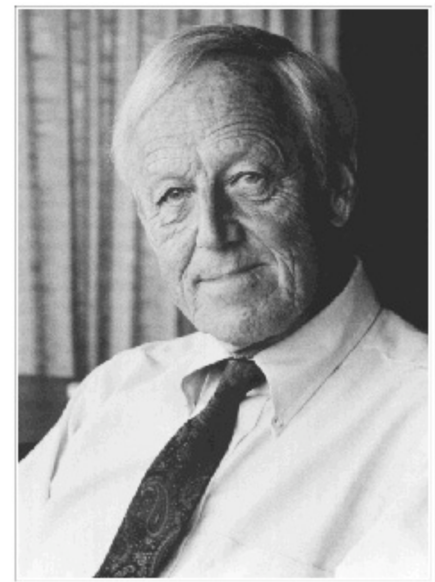
# Huge influence of Alfvén

- **Many contributions to plasma physics**
  - Existence of electromagnetic-hydromagnetic (“Alfvén”) waves (1942)
  - Concepts of guiding center approximation, first adiabatic invariant, frozen-in flux
  - Acceleration of cosmic rays (→ Fermi acceleration)
  - Field-aligned electric currents in the aurora (double layer)
  - Stability of Earth-circulating energetic particles (→ Van Allen belts)
  - Effect of magnetic storms on Earth’s magnetic field
  - Alfvén critical-velocity ionization mechanism
  - Formation of comet tails
  - Plasma cosmology (Alfvén-Klein model)
  - Books: *Cosmical Electrodynamics* (1950), *On the Origin of the Solar System* (1954), *World-Antiworlds* (1966), *Cosmic Plasma* (1981)
- **Wide-spread name**
  - Alfvén wave, Alfvén layer, Alfvén critical point, Alfvén radii, Alfvén distances, Alfvén resonance, ….
  - European Geophysical Union Hannes Alfvén Medal; European Physical Society (Plasma Physics Division) Hannes Alfven Prize




# Factoids

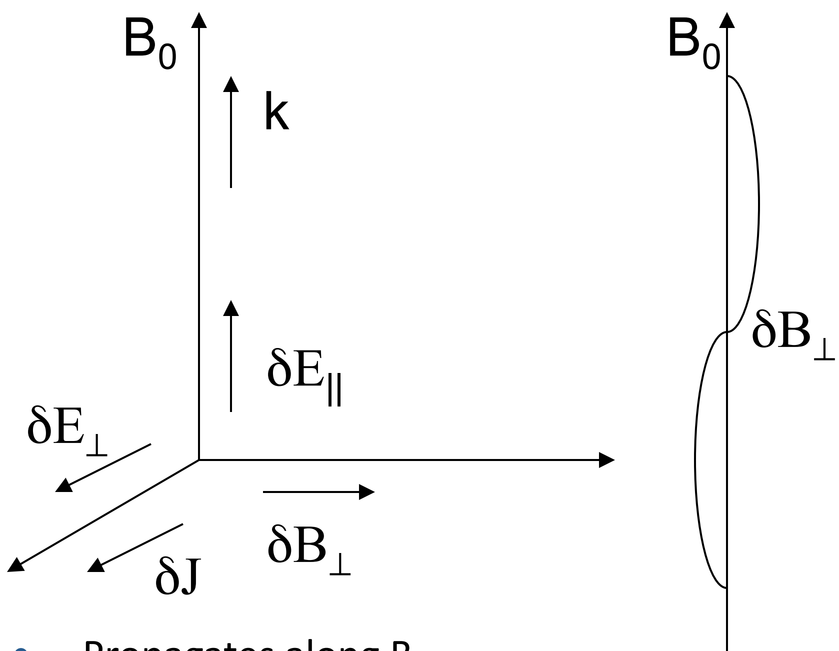
- His youthful involvement in a radio club at school later led (he claimed) to his PhD thesis on "Ultra-Short Electromagnetic waves"
- He had difficulty publishing in standard astrophysical journals (due to disputes with Sidney Chapman)
  - Fermi: "Of course" (1948)
- He considered himself an electrical engineer more than a physicist
- He distrusted computers
- The asteroid "1778 Alfvén" was named in his honor
- He was active in international disarmament movements
- The music composer Hugo Alfvén was his uncle



# *ALFVÉN WAVES*

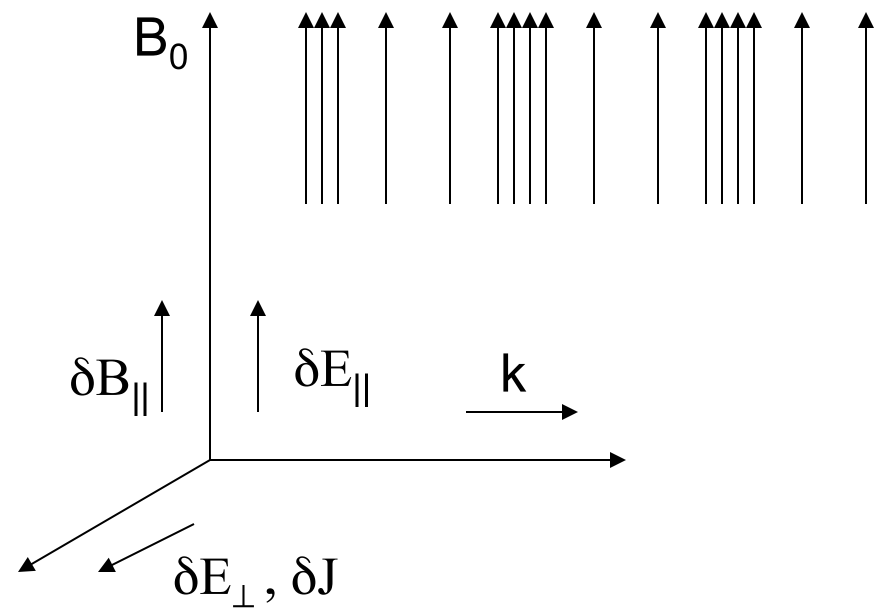
# Two general types of Alfvén waves

**Shear Alfvén** $(k \| B_0,\ \delta B_{\|} \cong 0)$




- Propagates along $B_0$
- Oscillation resembles a plucked violin string (i.e., driven by $B_0$-line tension)

**Compressional Alfvén** $(k \perp B_0,\ \delta B_\perp \cong 0)$




- Propagates across $B_0$
- Compression-rarefaction wave (i.e., driven by magnetic/plasma pressure)
- Higher frequency, since $k_\perp >> k_{\|}$

# Shear Alfvén continuum

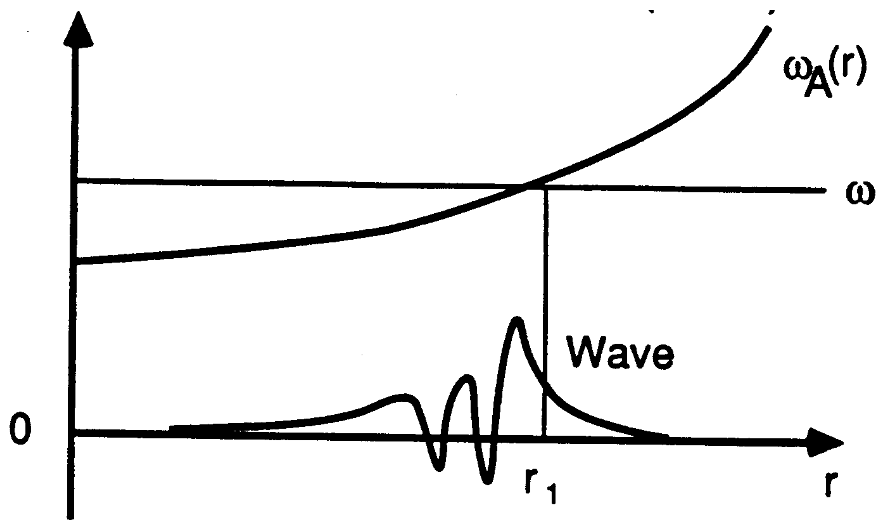
- **Ideal-MHD eigenmode equation (Hain-Lüst Eqtn) for cylindrical or large-aspect-ratio toroidal geometry**:

$$(\ldots)\frac{d\xi_r^{\,4}}{dr^4}+\frac{d}{dr}\left\{\left[\frac{\omega^2-k_\parallel^{\,2}v_A^{\,2}}{\omega^2-\left(k_\perp^{\,2}+k_\parallel^{\,2}\right)v_A^{\,2}}\right]\frac{B^2}{r}\frac{d}{dr}(r\xi_r)\right\}+\rho\left[\left(\omega^2-k_\parallel^{\,2}v_A^{\,2}\right)+\ldots\right]\xi_r=0$$

- **Coefficient of $d^2\xi_r/dr^2$ vanishes when $\omega^2 = k_{||}^2 v_A^2$ (shear Alfvén continuum)**
  - The mode structure is singular when the frequency satisfies the inequality Min $(k_{||}^2 v_A^2) \le \omega^2 \le$ Max $(k_{||}^2 v_A^2)$
  - Alfvén velocity is a function of radius in an inhomogeneous plasma:

$$v_A(r)=\frac{B_0(r)}{\sqrt{4\pi n_0(r)M_i}}$$

# Kinetic Alfvén Wave (KAW)

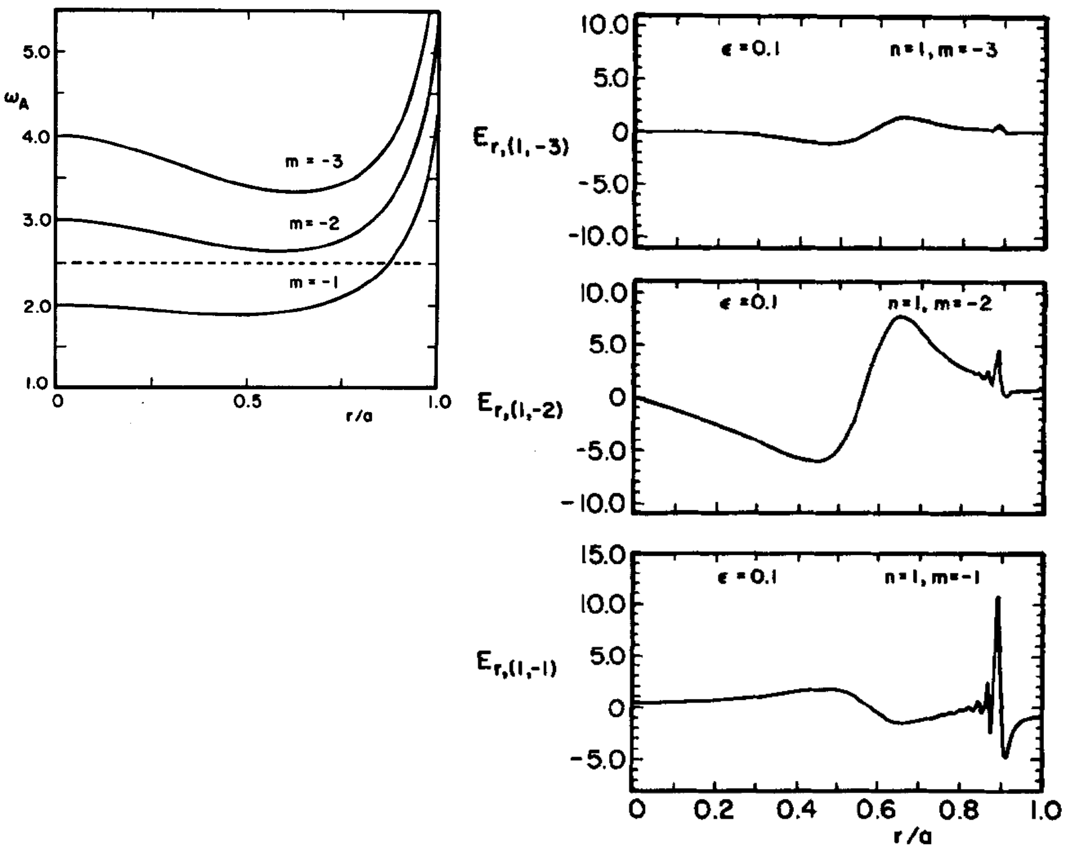



$$\omega_A(r) = k_\parallel v_A(r) \propto \frac{1}{\sqrt{n_0(r)}}$$

- **Solution is singular at position ($r_1$) of local Alfvén resonance where $\omega = \omega_A(r)$**
  - Resonant absorption of wave energy ("continuum damping")

- **If electron parallel dynamics and ion FLR effects are included, a non-singular solution can be obtained: "Kinetic Alfvén Wave"**
  - However, KAW experiences strong bulk plasma Landau damping, due to its short wavelength
  - Hence, the global-type Alfvén waves (GAE and TAE) are of more interest, since they have $\omega \neq \omega_A(r)$

# Global Alfvén Eigenmode (GAE)




$$E_r(r,\theta,\zeta) = \sum_m E_{r,(m,n)}(r)\exp(im\theta - in\zeta)$$

- **GAE is a radially extended, regular, spatially non-resonant discrete Alfvén eigenmode**
  - Requires that the current profile be such that the Alfvén continuum have an off-axis minimum ($k_{||} \neq 0$, thus nm<0):

$$\frac{d}{dr}\omega_A(r_1) = 0 \;\Rightarrow\; \frac{1}{k_{||}}\frac{dk_{||}}{dr} = -\frac{1}{v_A}\frac{dv_A}{dr}$$

  - Frequency lies just below the lower edge of the continuum
  - Sidebands suffer continuum damping
  - Experiments tried to use GAE for "global" tokamak plasma heating (Texas, Lausanne)

# *TOROIDAL ALFVÉN EIGENMODE*

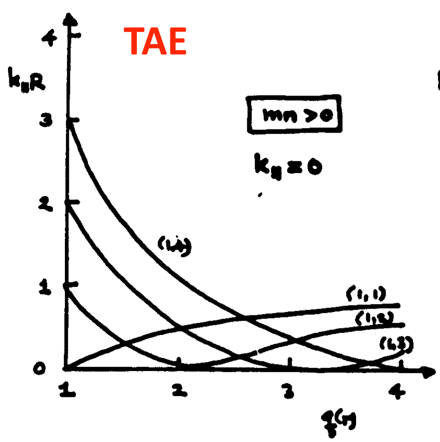
# Alfvén waves in toroidal geometry

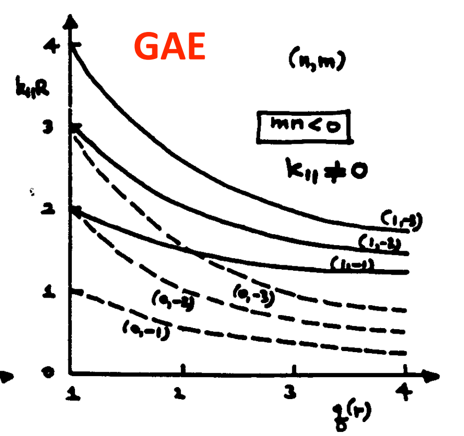
- **In a torus, wave solutions are quantized poloidally & toroidally:**

$$\Phi(r,\theta,\zeta,t) = \exp(-i\omega t)\sum_m \Phi_m(r)\exp(im\theta - in\zeta)$$

- **Parallel wave number $k_{||}$ determined by B-line twist $q(r)= rB_T/RB_p$ ("safety factor"):**

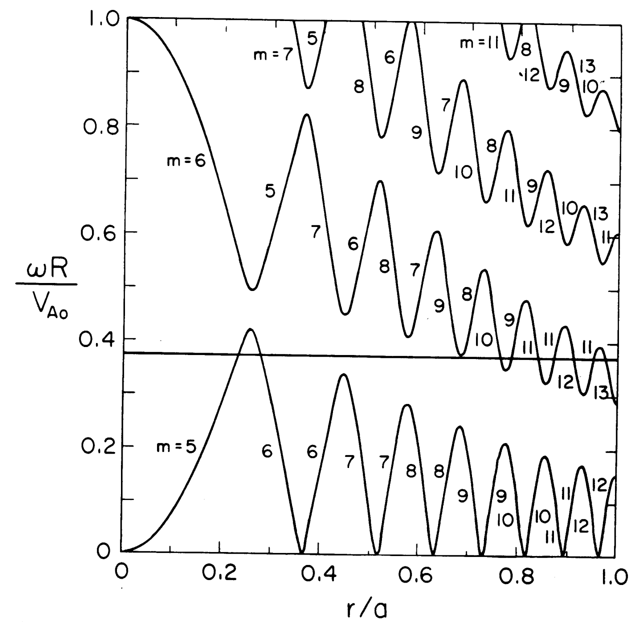
$$k_\parallel = \frac{1}{R}\left(\frac{m}{q(r)} - n\right)$$



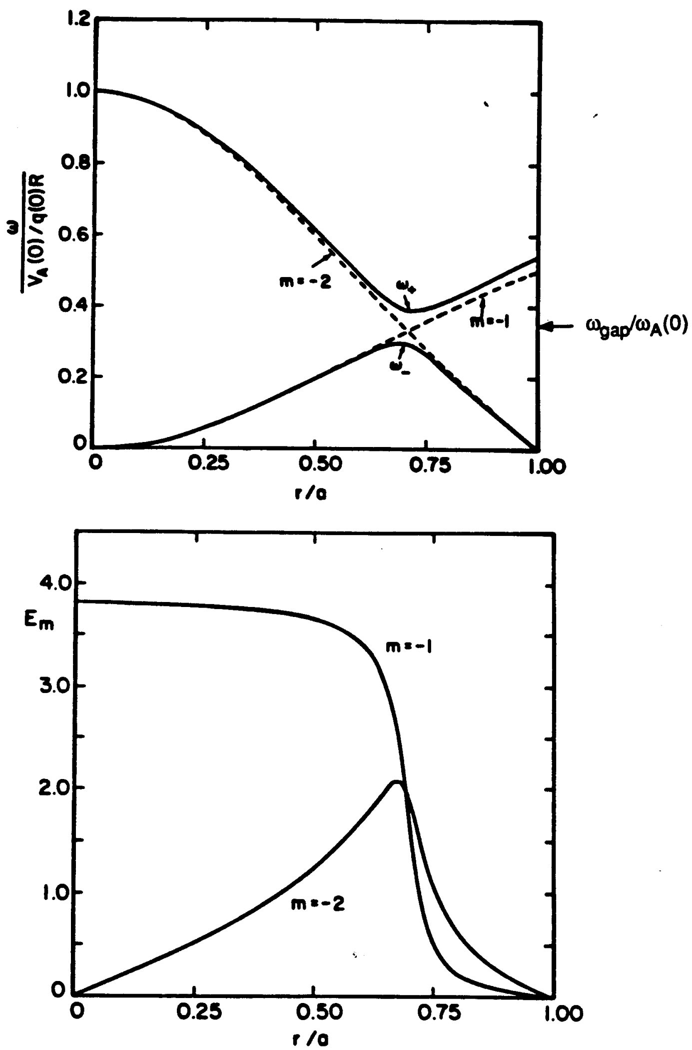

- **"Gaps" occur in Alfvén continuum in toroidal geometry when**

$$\omega = k_{\parallel m} v_A(r) = -k_{\parallel m+1} v_A(r)$$




- **Discrete eigenmodes exist within gaps due to equilibrium poloidal dependence:** e.g., $B_0 \propto 1 - (r/R)\cos\theta$

# Toroidal Alfvén Eigenmode (TAE)




- **TAE is a discrete, radially extended, regular (non-resonant) eigenmode**
  - Frequency lies in "gap" of width ~r/R <<1 at q=(m+1/2)/n, formed by toroidicity-induced coupling (m±1)

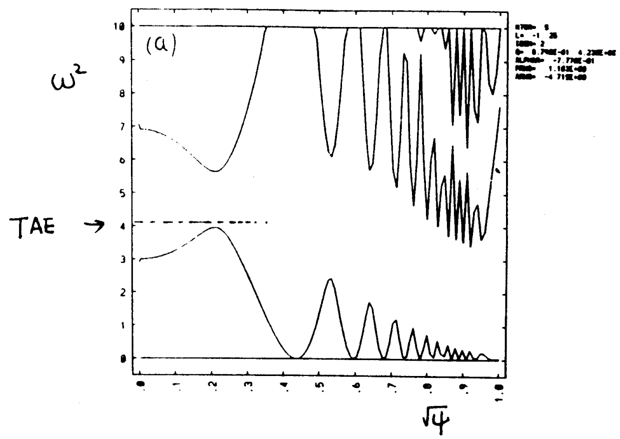
$$\omega_{TAE}^{m,m+1} = \left(\frac{n}{2m+1}\right)\frac{v_A}{R} \neq \omega_A(r)$$

  - Analogy to band gap theory in solid-state crystals (Mathieu equation, Bloch functions): "fiber glass wave guide"

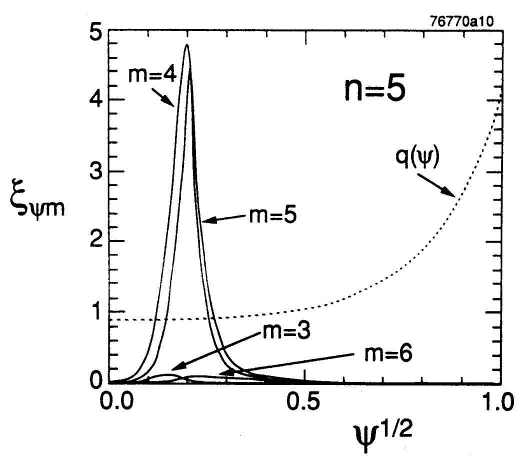
- **Similarly there exist:**
  - Ellipticity-induced Alfvén eigenmode (EAE): m±2
  - Triangularity-induced Alfvén eigenmode (NAE): m±3

# Core-localized TAE

- **At low shear (near magnetic axis or near internal transport barrier), the TAE moves near the bottom of the gap**
  - Theoretical explanation requires retaining higher-order finite aspect ratio effects




- **In addition, a second core-localized TAE appears at the top of the gap**



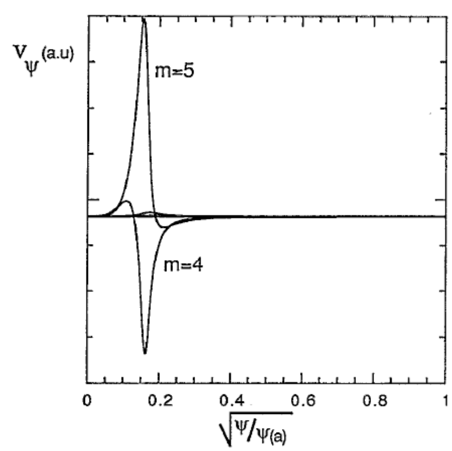




FIG. 5. Eigenfunction of the upper core-localized TAE, calculated from the CASTOR code: $n=5$ with dominant poloidal harmonics $m=4$ and 5; eigenfrequency $\omega=0.5951 v_A(0)/R$.

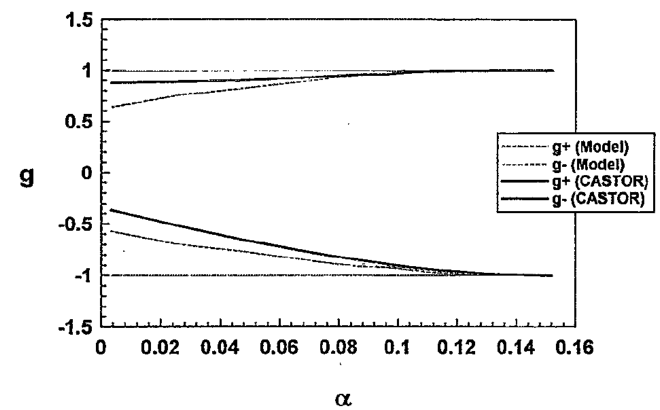



FIG. 6. Upper and lower core-localized TAE eigenfrequencies as calculated from the CASTOR code (thin solid curves) and from integration of the model equations (dashed curves), compared with the CASTOR-calculated upper and lower Alfvén continua (thick solid curves), as functions of the plasma pressure gradient $\alpha$.

# *TOROIDAL ALFVÉN EIGENMODE*
# *— Theoretical Derivation —*

# Eigenmode equations

- **Begin with:**
  - Charge neutrality $\nabla \cdot \vec{J} = 0$
  - Momentum balance $\rho_i \dfrac{d\vec{v}_i}{dt} = \dfrac{1}{c}\left(\vec{J} \times \vec{B}\right) - \nabla \cdot \vec{P}$
  - Maxwell's equations $\nabla \times \vec{B} = (4\pi / c)\vec{J}$ , $\nabla \times \vec{E} + (1/c)(d\vec{B}/dt) = 0$
  - Pressure equation: either a fluid equation of state (e.g., $P\rho^{-\gamma}$ = const.) or a kinetic equation (Vlasov, gyrokinetic, drift kinetic)

- **Linearize equations, with choice for field variables:**
  - Perturbed E-field components (usually $E_r$, $E_\theta$, $E_{||}$): useful for RF heating and antenna problems
  - Plasma displacement $\xi$ (where $d\xi/dt$ = **v**): useful for ideal MHD ($E_{||}$=0)
  - Potentials $\phi$, **A** (where $\vec{E} = -\nabla\phi - (1/c)\left(\partial\vec{A}/\partial t\right)$ , $\vec{B} = \nabla \times \vec{A}$ : useful for solving kinetic equations (need to choose a gauge)

# Low-mode-number TAE equation

- **For TAE: take $E_{||}$ = 0 (MHD-like); assume low beta (so $B_{||} \approx 0$)**

- **Linearized equation:**

$$\vec{B}_0 \cdot \nabla\left(\frac{\vec{B}_0 \cdot \vec{J}}{B_0{}^2}\right) - \frac{i\omega c}{4\pi}\nabla \cdot \left(\frac{\vec{B}_0 \times \vec{v}}{v_A{}^2}\right) + \left(\vec{B}_\perp \cdot \nabla\right)\left(\frac{\vec{B}_0 \cdot \vec{J}_0}{B_0{}^2}\right) + c\nabla \cdot \left[\frac{\vec{B}_0 \times \left(\nabla \cdot \vec{P}\right)}{B_0{}^2}\right] = 0$$

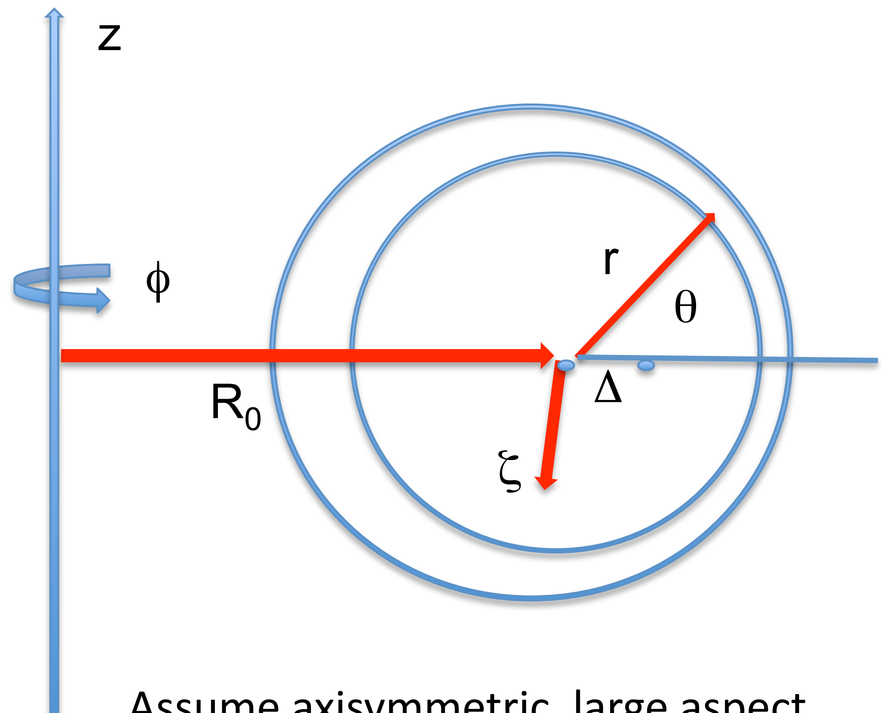
- **Terms**
  - 1[st] term (line bending): For low β, only $A_{||}$, so $\left(\frac{4\pi}{c}\right)\vec{B}_0 \cdot \vec{J} \cong -\nabla \cdot \left[B_0{}^2 \nabla_\perp \left(\frac{A_{\|}}{B_0}\right)\right]$
  - 2[nd] term (inertial): For ω << $\Omega_{ci}$, ion velocity $\vec{v} = c\left(\vec{E} \times \vec{B}_0\right)/B_0{}^2$
  - 3[rd] term (kink): For low beta, $\vec{B}_\perp \cong -\vec{B}_0 \times \nabla(A_{\|}/B_0)$
  - 4[th] term (kinetic): Use $\vec{v}_d = \frac{v_{\|}}{\Omega_c}\left(\nabla \times \hat{b} v_{\|}\right)$ and $\begin{Bmatrix} P_\perp \\ P_{\|} \end{Bmatrix} = M \int d^3v \begin{Bmatrix} v_\perp{}^2/2 \\ v_{\|}{}^2 \end{Bmatrix} f(\vec{r},\vec{v})$
    to obtain (for low beta):

$$c\nabla \cdot \left[\frac{\vec{B}_0 \times \left(\nabla \cdot \vec{P}\right)}{B_0{}^2}\right] \cong \sum_s e_s \int d^3v\, \vec{v}_{d,s} \cdot \nabla f_s$$

# Flux-type coordinates (1)

- **Various coordinate systems:**
  - Cylindrical (R, $\phi$, z) on center line
  - Shafranov coordinates on shifted flux surface ($r_s$, $\theta_s$, $\zeta_s$)
  - Flux-type ($r_f$, $\theta_f$, $\zeta_f$) for which field lines are "straight" (i.e., safety factor q is only a function of flux, not of $\theta$):

$$B_0 \cdot \nabla\Phi = \left(B_0 \cdot \nabla\theta\right)\left[\frac{\partial}{\partial\theta} + \left(\frac{B_0 \cdot \nabla\zeta}{B_0 \cdot \nabla\theta}\right)\frac{\partial}{\partial\zeta}\right]$$




Assume axisymmetric, large aspect ratio, low beta toroidal plasma with shifted circular magnetic surfaces

- **Construct flux coordinates:**
  - Take $r_f = r_s$ and $\zeta_f = \zeta_s$. Solve for $\theta_f$ using

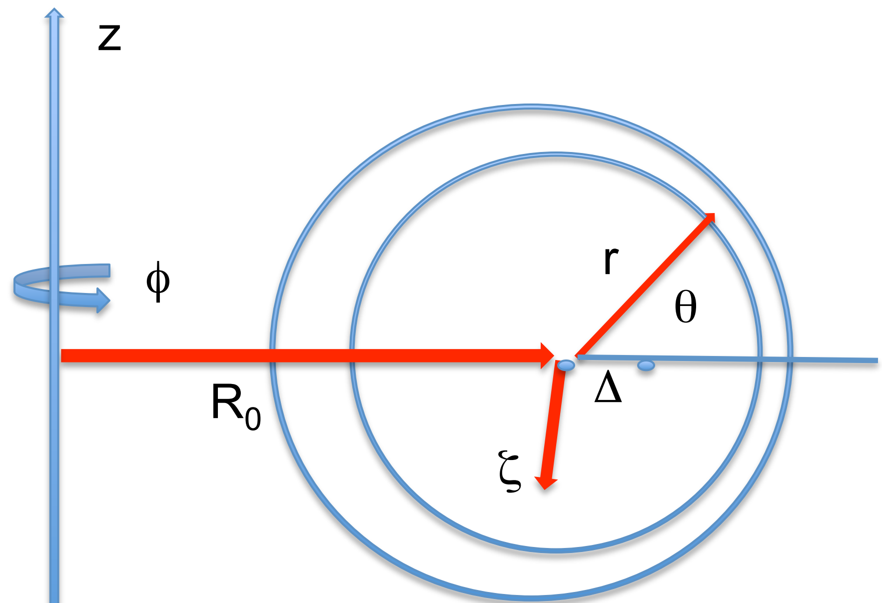
$$\frac{\partial}{\partial\theta}\left(\frac{B_0 \cdot \nabla\zeta}{B_0 \cdot \nabla\theta}\right) \equiv \frac{\partial q}{\partial\theta} = \frac{\partial}{\partial\theta}\left(\frac{J_f}{R_0^{\,2}}\right) = 0$$

$$J_s = rR_0\left[1 + \left(\frac{r - \Delta'}{R_0}\right)\cos\theta_s\right]$$

# Flux-type coordinates (2)

- **Shafranov coordinates:**

$$R = R_0 - \Delta(r_s) + r_s \cos\theta_s$$
$$\varphi = -\zeta_s$$
$$Z = r_s \sin\theta_s$$




- **Flux-type coordinates:**

$$R = R_0 - \Delta(r_f) + r_f \cos\theta_f + r_f \eta(r_f)\left(\cos 2\theta_f - 1\right)$$
$$\varphi = -\zeta_f$$
$$Z = r_f \sin\theta_f + r_f \eta(r_f) \sin 2\theta_f$$

$$\theta_f \cong \theta_s - \left(\frac{r}{R_0} + \Delta'\right)\sin\theta_s$$

– Here: $\eta(r) = \frac{1}{2}\left(\frac{r}{R_0} + \Delta'\right)$ with $\Delta' = \frac{d}{dr}\Delta \cong \frac{r}{R_0}\left(\beta_p + \frac{1}{2} l_i\right)$

# Large-aspect-ratio limit ($r/R_0 << 1$)

- **Expand equations, keeping terms up to $O(r/R_0)$**
  - Equilibrium magnetic field strength ~ $B_0\,[1 - (r/R_0)\cos\theta]$
  - Define "parallel wave number" $k_{||,m}(r) = [m/q(r) - n]/R_0$
  - Fourier decompose as $\Phi(r,\theta,\zeta) = \sum_m rE_m(r)\exp[i(m\theta - m\zeta)]$

- **Low-n TAE equation:**

$$\frac{d}{dr}\left[r^3\left(\frac{\omega^2}{v_A{}^2} - k_{\|m}{}^2\right)\frac{dE_m}{dr}\right] + r^2E_m\frac{d}{dr}\left(\frac{\omega^2}{v_A{}^2}\right) - \left(m^2-1\right)\left(\frac{\omega^2}{v_A{}^2} - k_{\|m}{}^2\right)rE_m$$

$$+\frac{d}{dr}\left[r^3\hat{\varepsilon}(r)\frac{\omega^2}{v_A{}^2}\left(\frac{dE_{m-1}}{dr} + \frac{dE_{m+1}}{dr}\right)\right] = -iL(\omega)E_m$$

  - L(w) represents kinetic (resonant) part of pressure response
  - $\hat{\varepsilon}(r) \approx \frac{5}{2}\left(\frac{r}{R_0}\right)$ represents toroidicity, which couples $E_m$ modes

# High-mode-number limit

- **Consider modes with $k_\perp >> k_\parallel$**
  - Hence, can ignore fast compressional Alfvén waves: $\nabla_\perp\left(4\pi p + \vec{B}_0 \cdot \vec{B}\right) \approx 0$

- **Use high-n ballooning mode representation**

$$\Phi(\psi,\theta,\zeta) = \sum_{l=-\infty}^{+\infty} \hat{\Phi}(\psi,\theta - 2\pi l,\zeta) \qquad \hat{\Phi} = \phi(\psi,\theta,\zeta)\, \exp\left[in\chi(q\theta - \zeta,\psi)\right]$$

  - Variable $\chi$ represents rapid cross-field variation $(B_0 \cdot \nabla\chi = 0)$ , and function $\phi$ the slow variation along field line on equilibrium scale
  - In the ballooning representation, $\theta$ is an "extended poloidal angle" with extent $-\infty < \theta < +\infty$
  - Express $\chi$ as $\chi = q\theta - \zeta + \int dq\, \theta_k(\psi)$ where $\theta_k$ is determined by a higher-order, radially nonlocal analysis
  - $\Phi$ is periodic (even though $\hat{\Phi}$ is not)

# High-n TAE equation

- **Canonical TAE equation in ballooning representation**

$$\frac{d^2}{d\theta^2}\psi + \left[\Omega^2\left(1 + 2\varepsilon\ \cos\theta\right) - \frac{s^2}{\left(1 + s^2\theta^2\right)^2}\right]\psi = 0$$

  - Toroidicity parameter $\varepsilon \approx r/R0 << 1$
  - Magnetic field shear $s = (r/q)(dq/dr)$
  - Normalized frequency $\Omega = \omega/\omega_A$, with Alfven frequency $\omega_A = v_A/qR_0$
  - Wave function: $\psi(\theta) = \frac{\Phi(\theta)}{\sqrt{1 + s^2\theta^2}}$

- **Solution is a mixture of secular and oscillatory behavior**
  - Require $\psi(\theta) \rightarrow 0\ ,\quad \theta \rightarrow \pm\infty$

# Existence of TAE frequency gap

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- **Examine asymptotic solution ($s\theta >> 1$)**
  - TAE equation has the form of the Mathieu equation

$$\left[\frac{d^2}{dz^2} + a - 2q\cos(2z)\right]\psi(z) \cong 0$$

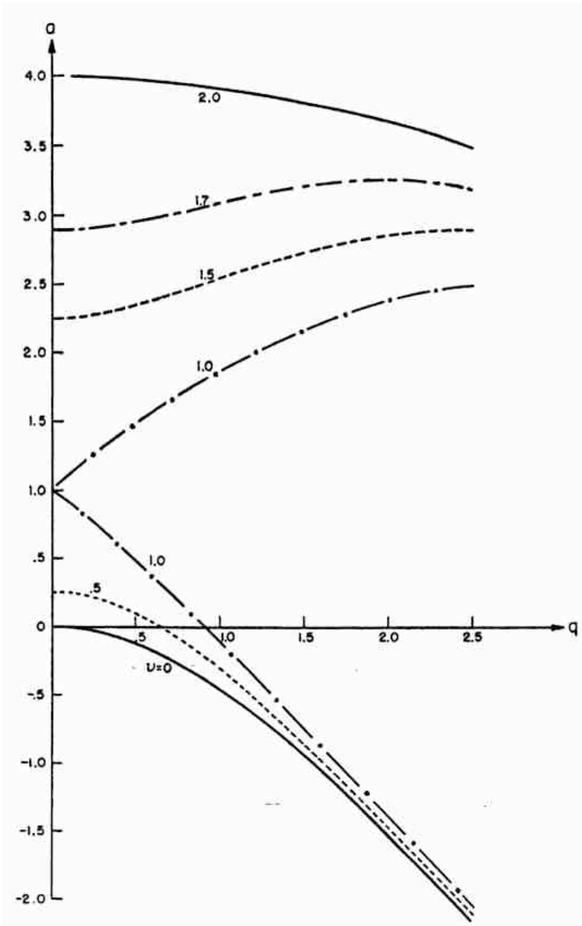
  - Therefore we know the solution has the Floquet form $\psi(z) = e^{\mu z} P(z)$, where P is periodic with period $\pi$ and the quantity $\mu$ is evaluated from Hill's determinant as

$$\cosh(\mu\pi) = 1 - 2\,\Delta(0)\left[\sin\left(\frac{\pi\sqrt{a}}{2}\right)\right]^2$$

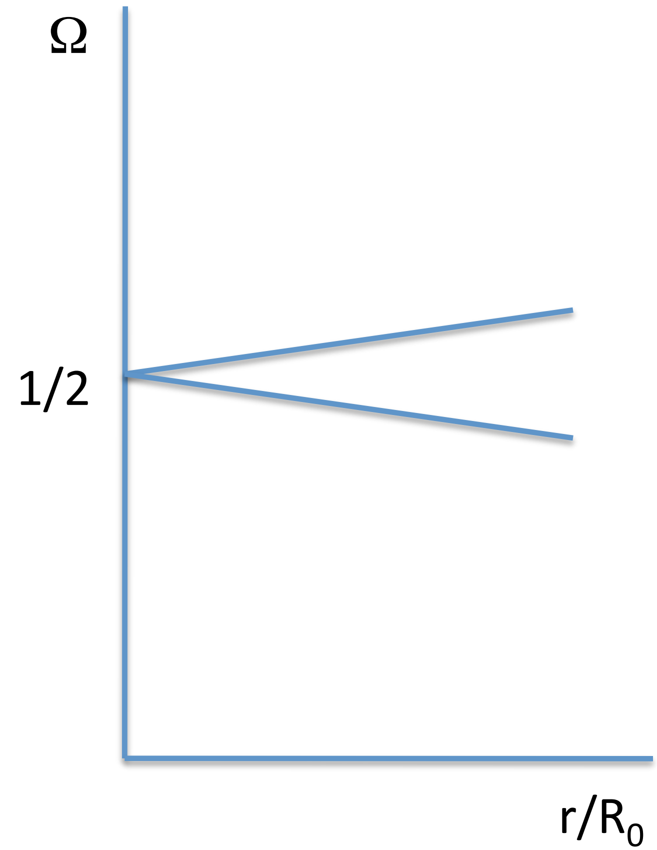
  - The quantity $\Delta(0) \cong 1 + \frac{\pi q^2}{(1-a)\sqrt{a}}\, ctn\left(\frac{\pi\sqrt{a}}{2}\right), \quad q << 1$

- **Focus near $\Omega$ = ½, where $\sin(2\pi\Omega)$ flips sign**
  - Find $|\cosh(\mu\pi)| < 1$ (unbounded solution) when $|\Omega\text{-}1/2| < (r/R_0)+\Delta'$, where $\Delta$ is Shafranov shift: gap of forbidden frequencies
  - Analogous to wave propagation in crystalline lattice (Brillouin bands)

# “Forbidden” frequency zone




**Matheiu equation unstable zones**




**TAE frequency gap**

# Existence of discrete eigenmode

- **It can be shown that within TAE gap, there is a discrete frequency at which the solution is well behaved**

$$\psi(z) \xrightarrow[z>>1]{} \left[A_+(\omega)e^{+\operatorname{Re}(\mu)z} + A_-(\omega)e^{-\operatorname{Re}(\mu)z}\right]e^{i\operatorname{Im}(\mu)z}P(z)$$

  - At some frequency $\omega = \omega_{TAE}$, the coefficient $A_+ = 0$

- **To demonstrate this, we need to solve for $A_+$**
  - Requires matching the z >> 1 solution to the z << 1 solution, since $A_+$ is a "slow" function of $z = \theta/2$

# Two-space-scale approach

- **Write** $$\psi(\theta) = \psi_c(\theta)\cos\left(\frac{\theta}{2}\right) + \psi_s(\theta)\sin\left(\frac{\theta}{2}\right)$$

- **Decompose TAE equation, retaining only θ/2 variation**

$$\left[\frac{d^2}{d\theta^2} + \left(\Omega^2 - \frac{1}{4}\right) + \varepsilon\Omega^2 - \frac{s^2}{\left(1+s^2\theta^2\right)^2}\right]\psi_c = -\frac{d\psi_s}{d\theta}$$

$$\left[\frac{d^2}{d\theta^2} + \left(\Omega^2 - \frac{1}{4}\right) - \varepsilon\Omega^2 - \frac{s^2}{\left(1+s^2\theta^2\right)^2}\right]\psi_s = \frac{d\psi_c}{d\theta}$$

- **In the x = sθ >> 1 regime, the well-behaved solution is**

$$\psi_c = A\exp\left[-\frac{x}{s}\sqrt{\left(\varepsilon\Omega^2\right)^2 - \left(\Omega^2 - \frac{1}{4}\right)}\right]$$

  – Frequencies for which $(\Omega^2-1/4)^2 - (\varepsilon\Omega^2)^2 < 0$ are in the TAE gap, whose boundaries are $\Omega^2 = ¼ \pm \varepsilon\Omega^2 \sim ¼(1\pm\varepsilon)$

# TAE dispersion relation

- **The large x solution ($x = s\theta >> 1$) can be asymptotically matched to the small x solution ($x \sim 1$)**
  - Analytically obtainable in either low shear ($\Omega/s >> 1$) or high shear ($\Omega/s << 1$) limit
  - Results in dispersion relation for the TAE

- **Example: low shear ($s << 1$)**

$$\Omega_{TAE}{}^2 \cong \frac{1}{4}\left\{1-\varepsilon\left[\frac{\left(1-\frac{\pi^2 s^2}{32}\right)-\frac{\pi^2 s^2}{16}}{\left(1-\frac{\pi^2 s^2}{32}\right)+\frac{\pi^2 s^2}{16}}\right]\right\} \approx \frac{1}{4}\left[1-\varepsilon\left(1-\frac{\pi^2 s^2}{8}\right)\right]$$

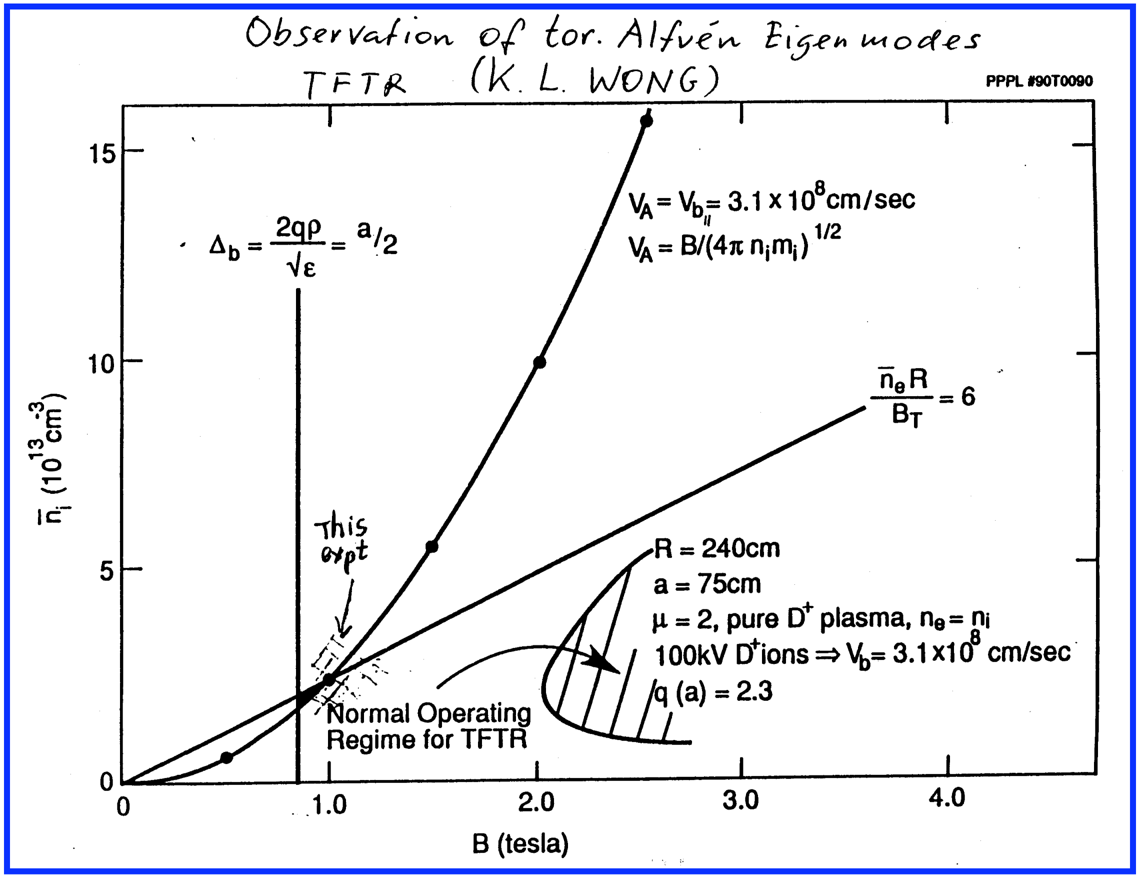
- **If repeat the derivation with kinetic resonance term included, $\Omega_{TAE}$ becomes complex, giving the growth rate**
  - Further calculations yield TAE continuum damping

# TAE instability

- **Theoretical growth rate:**

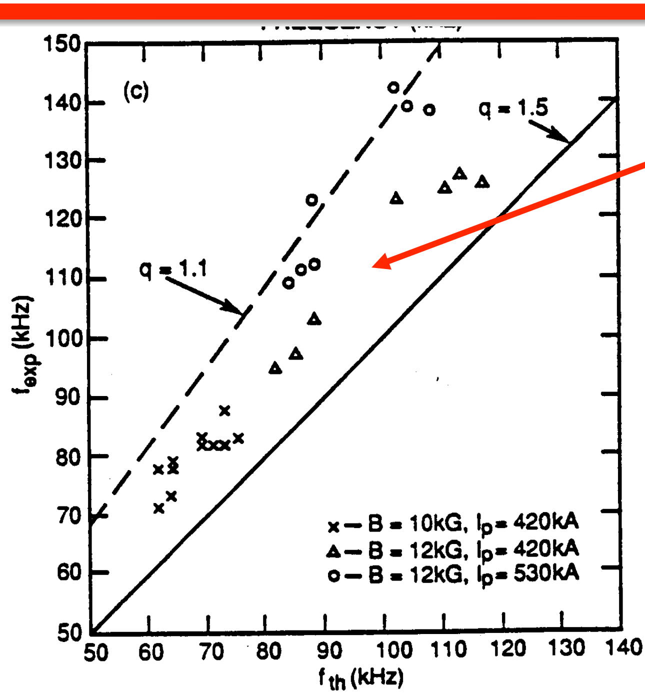
$$\frac{\gamma}{\omega} \cong \frac{9}{4}\left[\beta_\alpha\left(\frac{\omega_{*\alpha}}{\omega_0}-\frac{1}{2}\right)F-\beta_e\left(\frac{v_A}{v_e}\right)\right]$$

- **Instability requires:**
  - Wave-particle kinetic resonance ($v_a \geq v_A$)
  - Inverse damping ($\omega_{*\alpha} > \omega_0$)
  - Growth overcomes damping ($\beta_\alpha/\beta_e$ > “small number” -- for electron Landau damping; other damping mechanisms also important)




K. L. Wong et al., Phys. Rev. Lett. **66**, 1874 (1991)
W. W. Heidbrink et al., Nucl. Fusion **31**, 1635 (1991)

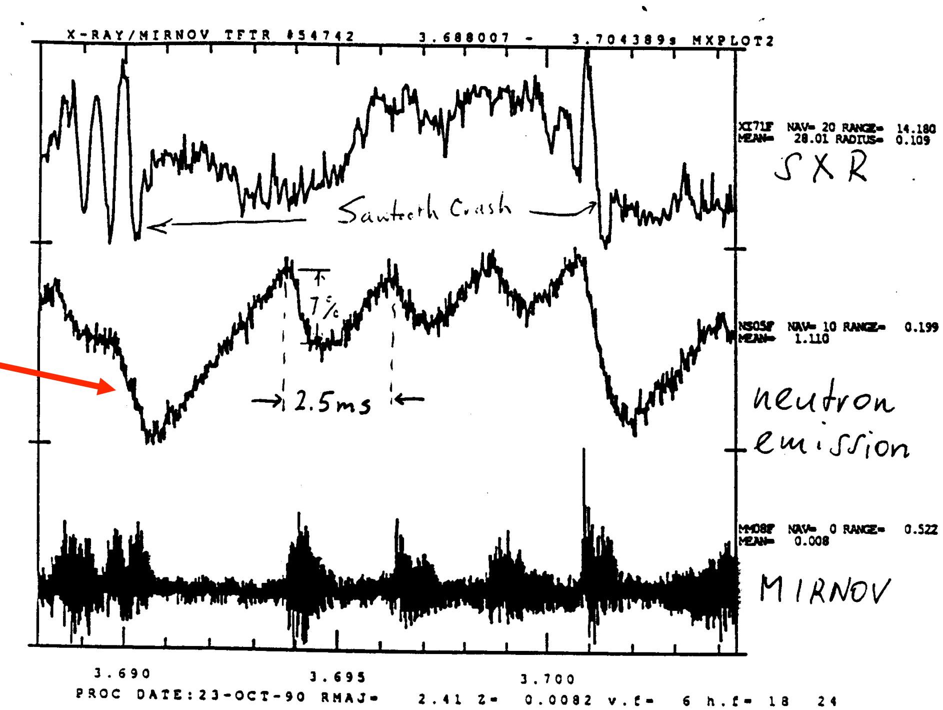
# TAE experimental observation




- Frequency scaled linearly with B
- Fluctuation amplitude increased with beam power

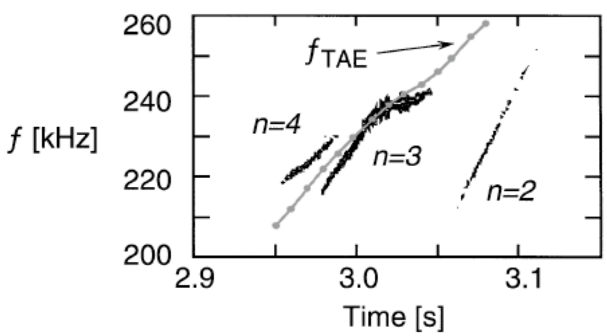
- Energetic beam ions were ejected: X-rays dropped 7% in periodic bursts

- A later experiment found intense fast ion fluxes that damaged vacuum vessel (ripple trapping caused by TAE resonance)



# Observation of α-driven TAE

- **DT discharge in TFTR**
  - Reduced TAE damping by observing after turn off of beam heating
  - Looked for low-shear core localized TAE, guided by theory




R. Nazikian, G. Y. Fu, et al., Phys. Rev. Lett. **78**, 2976 (1997)

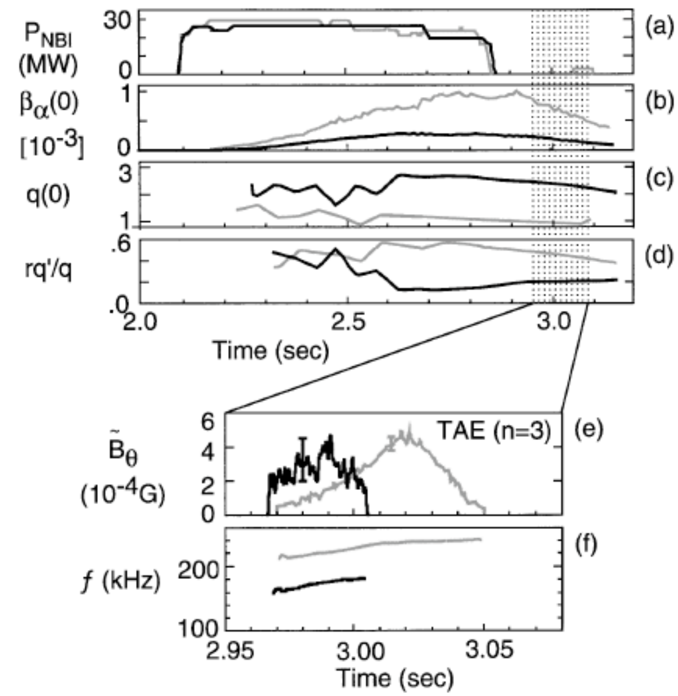
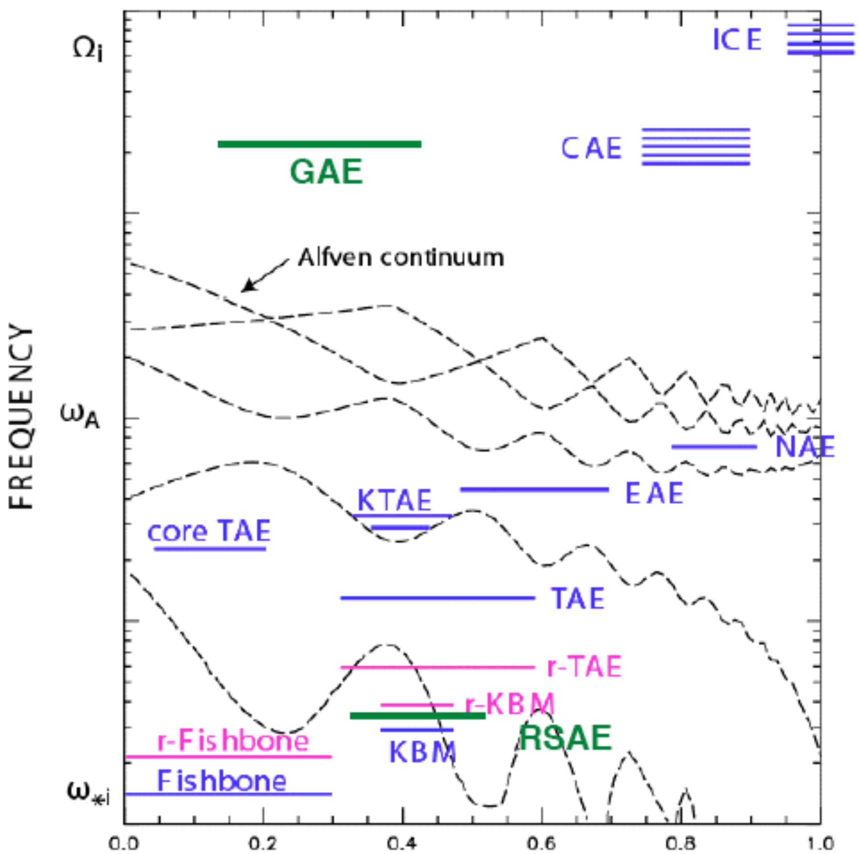



FIG. 1. Evolution of (a) neutral beam power, (b) central $\beta_\alpha$ (0), (c) central safety factor, (d) magnetic shear at $r/a \approx 0.3$, (e) external magnetic fluctuation amplitude, and (f) measured mode frequency for high and low $q(0)$ plasmas [indicated by black (gray) lines] corresponding to the following plasma parameters at the time of peak mode amplitude: $R =$ 260 cm (252 cm), $I_p$ = 1.6 MA (2.0 MA), $B_T$ = 5.3(5.1) T, $n_e(0) = 3.3(4.0) \times 10^{13}$ cm$^{-3}$, $T_i(0)$ = 11(15) keV, $T_e(0)$ = 5.4(6.0) keV.

# Zoology of *AE modes



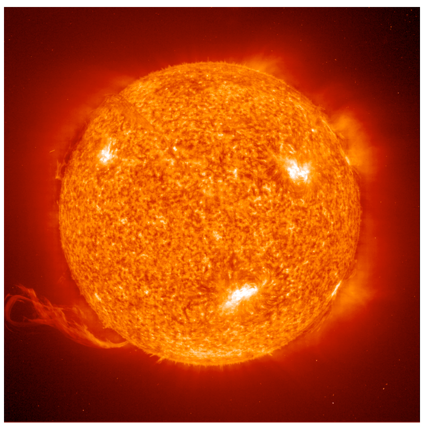

*Heidbrink, Phys. Pl. 9 (2002) 2113*

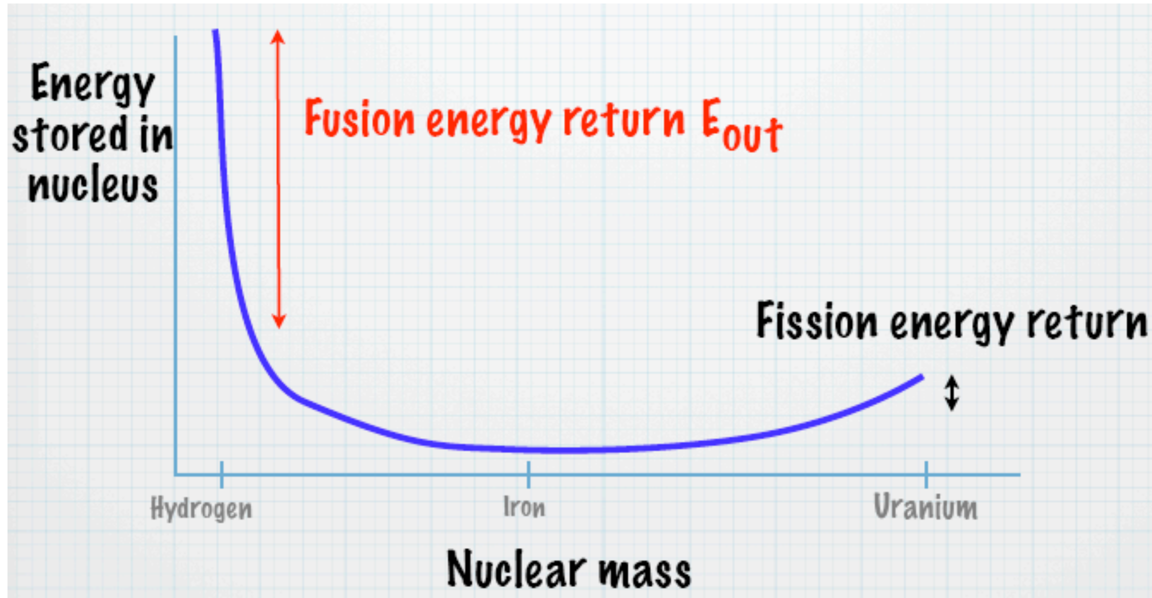
- **Fast particles can destabilize a large variety of Alfvén modes (*AE)**
  - e.g., Toroidal Alfvén Eigenmode (TAE)
- **Mode identification is robust:**
  - Frequency, mode structure, polarization
- **Threshold is determined by balance of:**
  - Growth rate (reliably calculate)
  - Damping rate (calculation is very sensitive to parameters, profiles, length scales—but can measure with active/ passive antennas)
- **Also, Energetic Particle Modes (EPM)**
  - Exist only in presence of energetic particles (e.g., NBI or RF ions, alphas)

# *TAE IMPLICATIONS FOR BURNING PLASMAS*

# What is a "burning plasma"?

Sun






- **"Burning" plasma** = ions undergo thermonuclear fusion reactions, which supply self-heating to the plasma

- The energy output $E_{out}$ is huge (global implications):

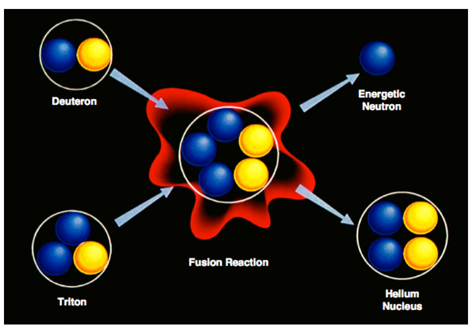
  **$E_{out}$ = 450 x $E_{in}$**

- The required energy input $E_{in}$ is also large:

  **20 keV = 200 million °K**

# D-T fusion

- **The "easiest" fusion reaction uses hydrogen isotopes: deuterium (D) and tritium (T)**
  – D is plentiful in sea water
  – T can be generated from lithium
  – He is harmless (even useful)

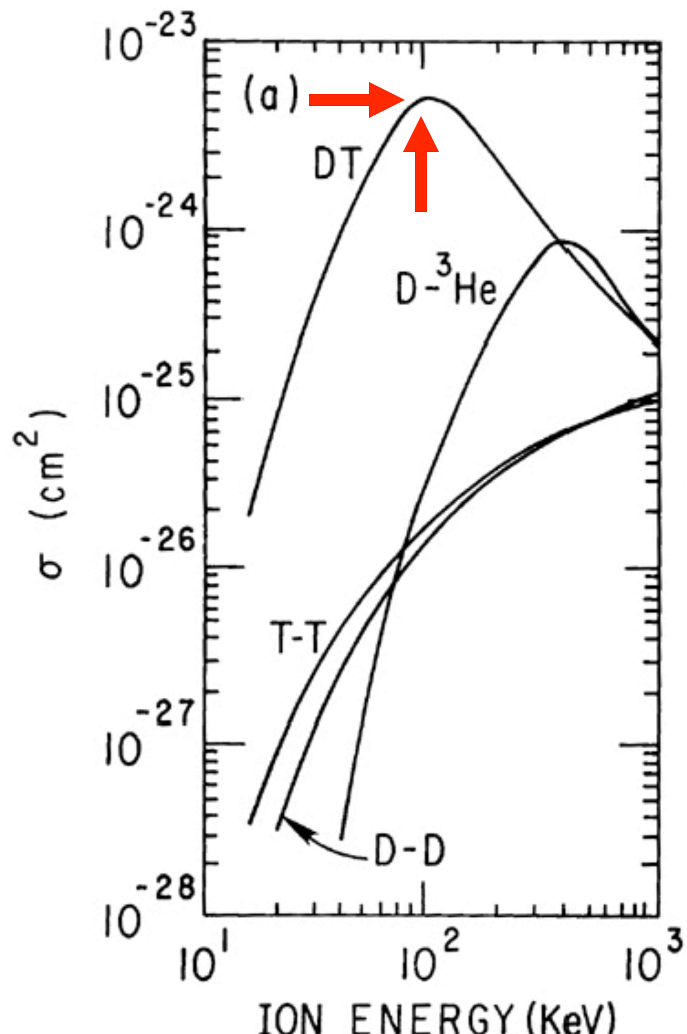



$${}_1D^2 + {}_1T^3 \rightarrow {}_2He^4 + {}_0n^1$$

(3.5 MeV) (14.1 MeV)

Energy/Fusion: $\varepsilon_f = 17.6$ MeV

**Nuclear cross sections**

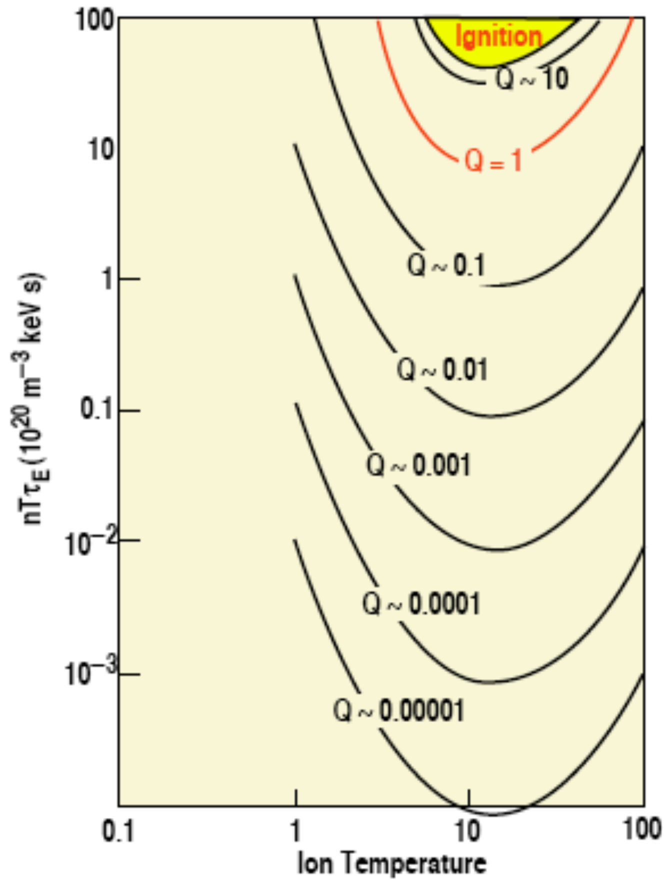


# Fusion gain Q

$$\frac{dW}{dt} \to 0 \implies P_\alpha + P_{heat} = \frac{W}{\tau_E}$$

Define fusion energy gain, $Q \equiv \dfrac{P_{fusion}}{P_{heat}} = \dfrac{5\,P_\alpha}{P_{heat}}$

Define $\alpha$-heating fraction, $f_\alpha \equiv \dfrac{P_\alpha}{P_\alpha + P_{heat}} = \dfrac{Q}{Q+5}$



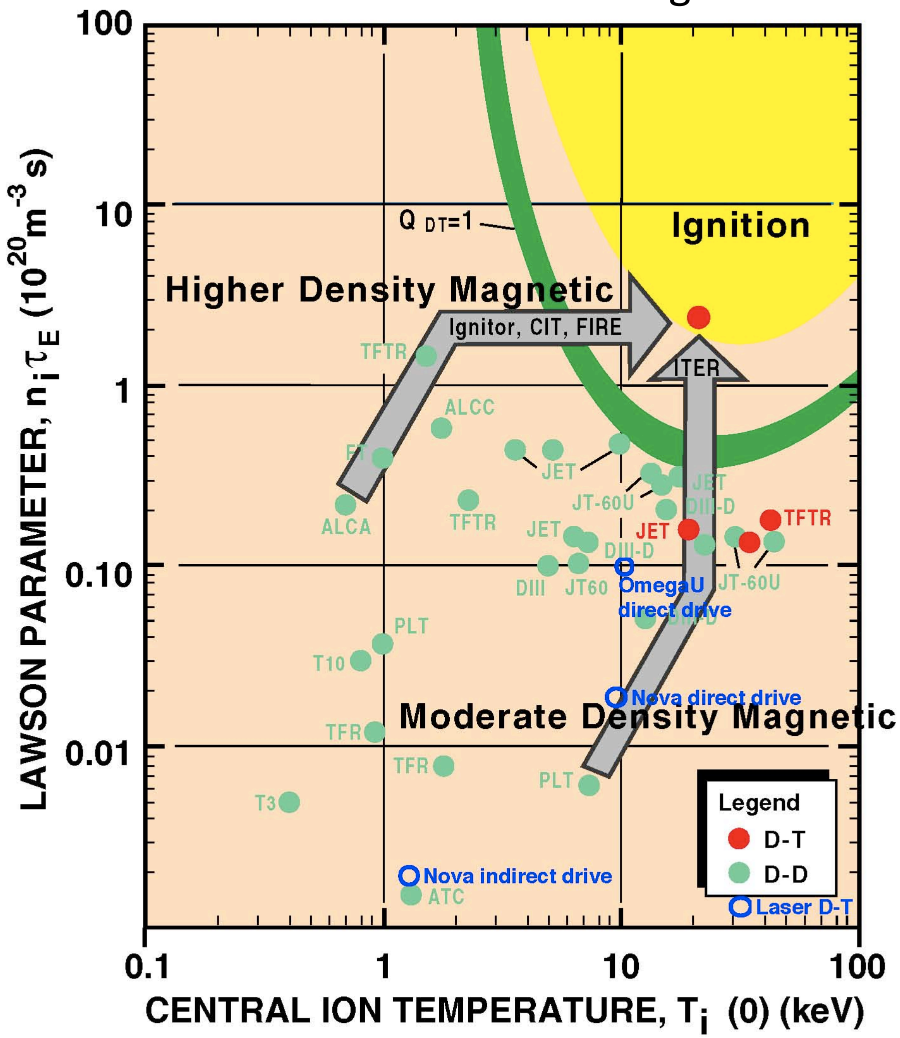

| | | |
|---|---|---|
| **Breakeven** | $Q = 1$ | $f_\alpha = 17\%$ |
| **Burning plasma regime** ↓ | $Q = 5$ | $f_\alpha = 50\%$ |
| | $Q = 10$ (ITER) | $f_\alpha = 60\%$ |
| | $Q = 20$ | $f_\alpha = 80\%$ |
| | $Q = \infty$ | $f_\alpha = 100\%$ |

# Burning plasmas: the next frontier

- **Status of magnetic fusion**
  - Achieved $T_i$ required for fusion, but need ~10 x n $\tau_E$
  - Achieved n $\tau_E \approx$ ½ required for fusion, but need ~10 x $T_i$

- **Understanding burning plasmas is today's fusion research challenge**
  - Necessary step forward on the path to fusion energy
  - World fusion program is technically and scientifically ready to proceed with a burning plasma experiment (➔ITER)

Lawson diagram



# Science challenges for burning plasmas

- **Many of the scientific challenges for burning plasmas are the <u>same</u> as those of today's experiments, albeit extended to new parameter ranges**
  - Plasma **equilibrium**
  - Macroscopic **stability**
  - **Transport** and confinement
  - **Supra-thermal particles** and **plasma-wave interactions**
  - **Measurement and control** tools

- **Burning plasmas also have <u>new</u> challenges**
  - Dynamics of exothermic medium
  - Self-heated and increasingly self-organized
  - Large plasma size
  - Large population of highly energetic alpha particles
  - Thermonuclear environment

# New science issues for burning plasmas

USBPO U.S. BURNING PLASMA ORGANIZATION

## Uniquely BP issues

- ***Alpha particles***
  - Large population of supra-thermal ions
- ***Self-heating***
  - "Autonomous" system (self-organized profiles)
  - Thermal stability

## Reactor-scale BP issues

- ***Scaling with size & B field***
- ***High performance***
  - Operational limits, heat flux on plasma-facing components
- ***Nuclear environment***
  - Radiation, tritium retention, dust, tritium breeding

**All issues are strongly coupled/integrated**

# Energetic particles

- **In addition to thermal ions and electrons, plasmas often contain a supra-thermal species = "energetic particles"**
  - Highly energetic ($T_f >> T_i$)
  - Low density ($n_f << n_i$), but comparable pressure ($n_f T_f \cong n_i T_i$)

- **Energetic particles can be created from various sources**
  - Externally: ion/electron cyclotron RF heating or neutral beam injection —> high-energy "tails" of ions and electrons
  - Internally: fusion reaction alpha particles; runaway electrons

- **The plasma physics of energetic particles is of interest to:**
  - Laboratory fusion plasmas (alphas provide self-heating to sustain ignition)
    - Can excite various types of Alfvén instabilities (since $v_\alpha \sim v_A$)
    - Can be redistributed or lost, leading to reduced fusion heating, increased heat loading on walls, etc.
  - Space and astrophysical plasmas (e.g., proton ring in Earth's magnetosphere)
  - High-energy-physics accelerators (collective effects)

# Alpha particle characteristics

- **Plasma ions and electrons:**
  - $T_{i,e}$ ~ 10-20 keV
  - "Frozen-in" behavior to lowest order (MHD description)
  - Thermodynamic equilibrium (Maxwellian distribution)

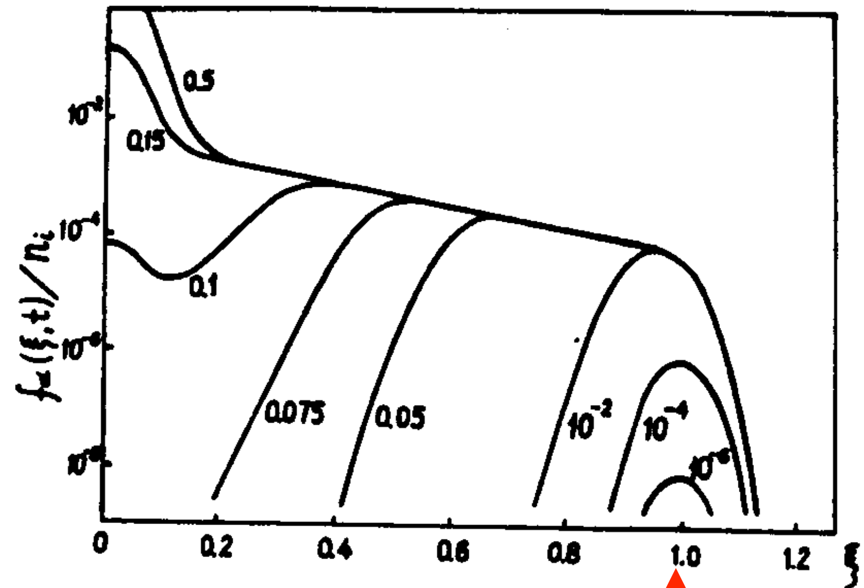
- **Alpha particles:**
  - High energy: $T_{\alpha,birth}^{DT}$ = 3.5 MeV
  - Not "frozen" to B-field lines (require kinetic description)
  - Low density ($n_{\alpha} < n_{i,e}$), but comparable pressure ($p_{\alpha} \sim p_{i,e}$)
  - Non-Maxwellian "slowing down" distribution
  - Centrally peaked profile

$$\left|\nabla p_{\alpha} / p_{\alpha}\right|^{-1} \leq a/2$$

- **Other energetic particles:**
  - Supra-thermal ions from NBI and ICRH
    - Can simulate $\alpha$-particle effects without reactivity (although NBI/ICRH ions are anisotropic in pitch angle, whereas alphas are isotropic)
    - Also present in burning plasmas with auxiliary heating
  - Run-away electrons associated with disruptions

# Birth, life, and death of alpha particles

- **DT alphas are born in peaked distribution at 3.5 MeV at rate $\partial n_\alpha/\partial t = n_D n_T \langle\sigma v\rangle$**
  - During time $\tau_s$, they are slowed down by collisions with electrons to smoother distribution at ~ 1 MeV
  - After time $\tau_M$, they thermalize against both electrons and ions to the plasma temperature ($T_e \sim T_i \sim 10$ keV)
  - Alphas are confined for time $\tau_\alpha$. In steady-state there are two alpha populations: slowing-down $\alpha$'s ($n_s$) and cool Maxwellian $\alpha$'s ($n_M$)
- **Typically $\tau_\alpha \sim 10\,\tau_M \sim 10^3\,\tau_s$ : hence $\alpha$'s have time to thermalize**
  - Since $n_s / n_\alpha \sim \tau_s / \tau_\alpha \sim 10^{-3}$, then $n_M \sim n_\alpha \sim n_e$ (for reactors); hence "ash" (slow $\alpha$'s) is a problem in reactors, because it will "poison" the plasma




Birth velocity:

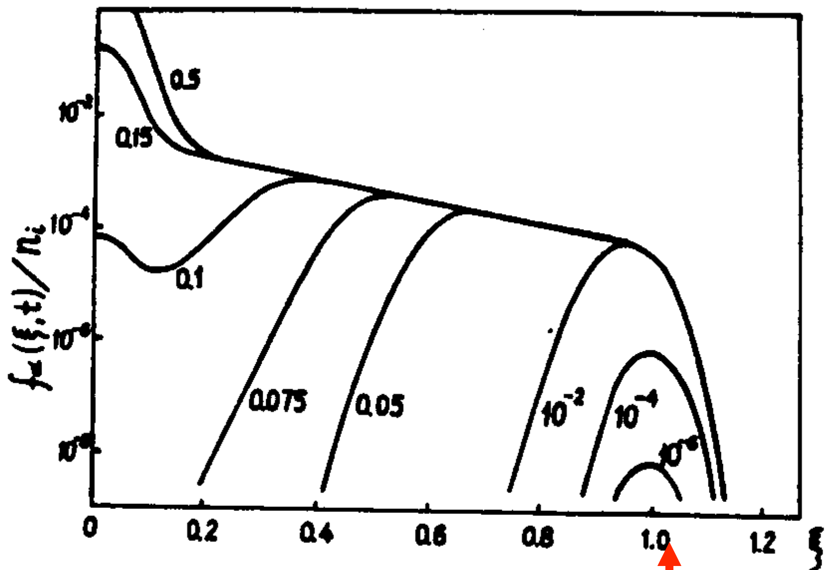
$$v_{\alpha 0}^{D-T} = 1.3 \times 10^9\, cm/s$$

$$P_{fus} \propto n^2 \langle \sigma v \rangle \sim n^2 T^2 \propto p^2$$

# Slowing-down distribution

- The classical steady-state "slowing down" distribution (isotropic) is

$$F_s(r,v) = \begin{cases} S(r)\tau_s / 4\pi\left(v^3 + v_c^{\ 3}\right), & v < v_{\alpha 0} \\ 0, & v > v_{\alpha 0} \end{cases}$$




Slowing-down time:

$$\tau_s = \frac{3 m_e m_\alpha v_e^{\ 3}}{16\sqrt{\pi} Z_\alpha^{\ 2} e^4 n_e \ln\Lambda_e} \cong 0.37\,\mathrm{sec}$$

Birth velocity:

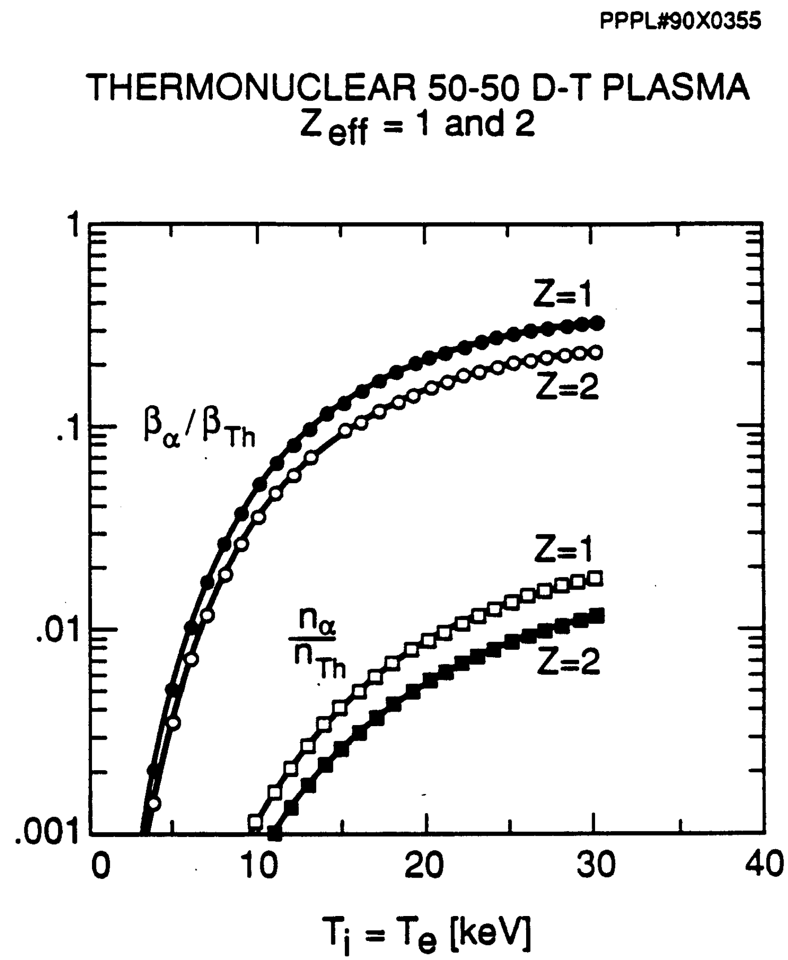
$$v_{\alpha 0}^{\ D-T} = 1.3\times 10^9\, cm/s$$

Critical velocity (balance ion/electron friction):

$$v_c = v_e \left[ \frac{3\sqrt{\pi} m_e}{4 m_p \ln\Lambda_e} \sum_i \frac{n_i Z_i^{\ 2} \ln\Lambda_i}{A_i n_e} \right]^{1/3} \cong 4.6\times 10^8\, cm/s$$

# Temperature dependence

- **Alpha parameters are determined by the plasma temperature**
  - For ~10 keV plasma, $\alpha$'s deposit their energy into thermal electrons, with slowing-down time $\tau_s \propto T_e^{3/2} / n_e$
  - Since the alpha source ~ $n_e^2 <\sigma v>$, we have $n_\alpha/n_e \sim T_e^{3/2} <\sigma v> \propto T_e^{3/2} T_i^2$
  - For an equal-temperature Maxwellian plasma, the ratios $n_\alpha/n_e$ and $\beta_\alpha/\beta_{plasma}$ are unique functions of temperature $T_e$.



# Significance for ITER

- **Approximately 200-600 MW of alpha heating needed to sustain ignition**
  - Huge amount of power to handle with no direct external control

- **Experimental relevance of alpha loss:**
  - Damage to first wall and divertor plate structure (wall loading)
  - Impurity influx
  - Reduced efficiency of current drive or heating
  - Operational control problems (e.g., thermal burn stability)
  - Quenching of ignition (e.g., fuel dilution by $\alpha$ ash)

- **Understanding of alpha physics is needed to:**
  - Assure good alpha confinement
  - Optimize alpha heating efficiency
  - Avoid alpha-driven collective instabilities (*AE modes)

- **Alpha dynamics integrated with overall plasma behavior**
  - Macro-stability, transport, heating, edge, …

# Parameter comparison

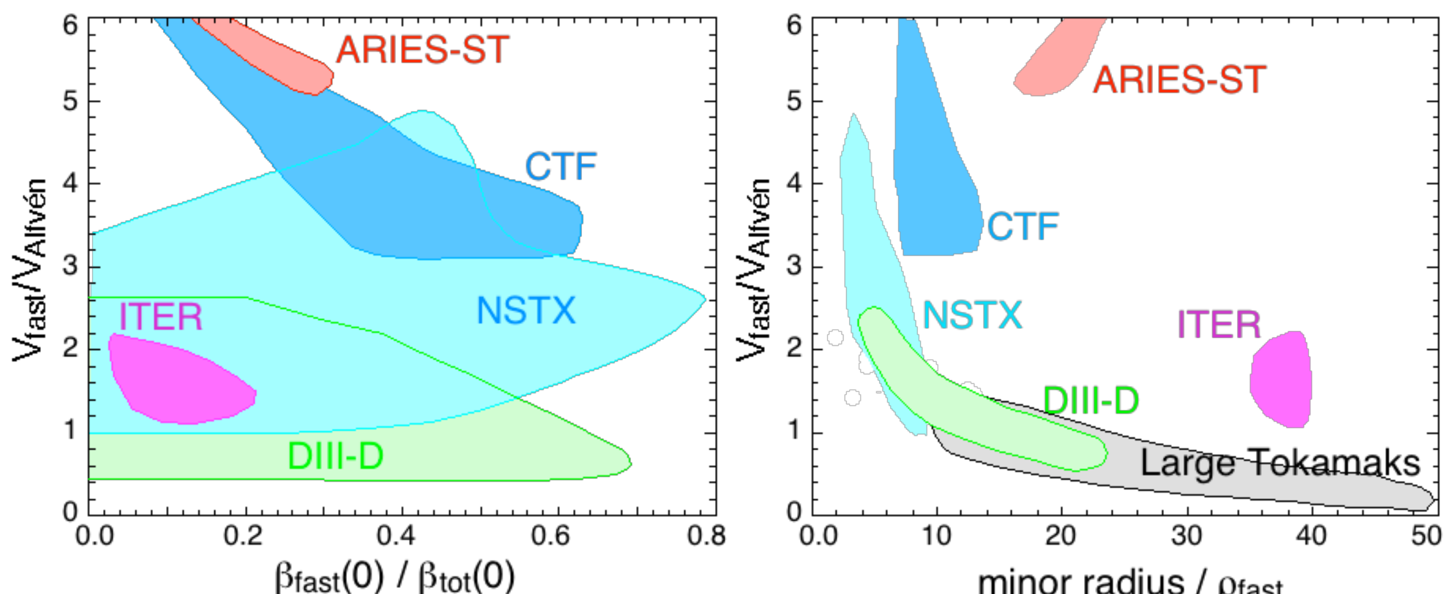
Fast ion parameters in contemporary experiments compared with projected ITER values.

| Tokamak | TFTR | JET | JT-60U | JET | ITER |
|---|---|---|---|---|---|
| Fast ion Source | Alpha Fusion | Alpha Fusion | Deuterium Co NBI | Alpha ICRF tail | Alpha Fusion |
| Reference | [3] | [3] | [34] | [20,52] | [52] |
| $\tau_S$ (s) | 0.5 | 1.0 | 0.085 | 0.4 | 0.8 |
| $\delta/a$[a] | 0.3 | 0.36 | 0.34 | 0.35 | 0.05 |
| $P_f(0)$ (MW m$^{-3}$) | 0.28 | 0.12 | 0.12 | 0.5 | 0.55 |
| $n_f(0)/n_e(0)$ (%) | 0.3 | 0.44 | 2 | 1.5 | 0.85 |
| $\beta_f(0)$ (%) | 0.26 | 0.7 | 0.6 | 3 | 1.2 |
| $\langle\beta_f\rangle$ (%) | 0.03 | 0.12 | 0.15 | 0.3 | 0.3 |
| max $\lvert R\nabla\beta_f\rvert$ (%) | 2.0 | 3.5 | 6 | 5 | 3.8 |
| $v_f(0)/v_A(0)$ | 1.6 | 1.6 | 1.9 | 1.3 | 1.9 |

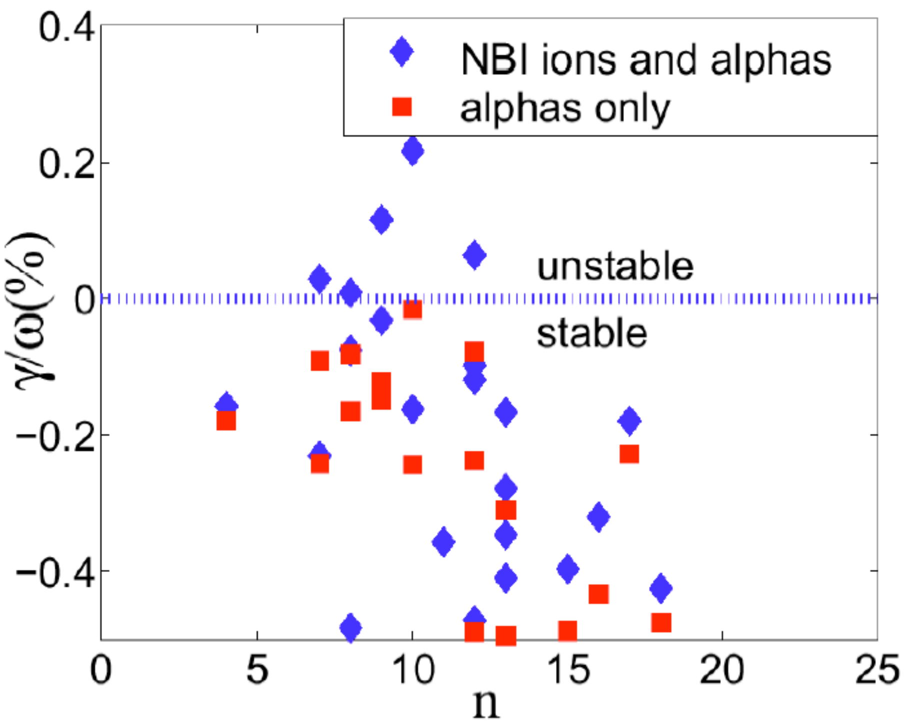
- **Differences for fast ("f") ion physics in ITER:**
  - Orbit size δ/a in ITER is much smaller
  - Most of the other parameters (especially dimensionless) are comparable
  - No external control of alphas, in contrast to NBI and ICRH fast ions

# TAE stability in ITER – 1

- **ITER will operate with a large population of super-Alfvénic energetic particles**
  - Alfvén Mach number ($v_\alpha/v_A$) and pressure ($\beta_\alpha$) for ITER alpha particles have similar values as in existing experiments
- **ITER's large size (and hence small-wavelength regime $\rho_{*fast}^{-1} = a/\rho_{fast} >> 1$) implies a "sea" of many potentially unstable TAE modes**
  - Could cause redistribution or loss of alpha particles ("domino" effect)



# TAE stability in ITER – 2

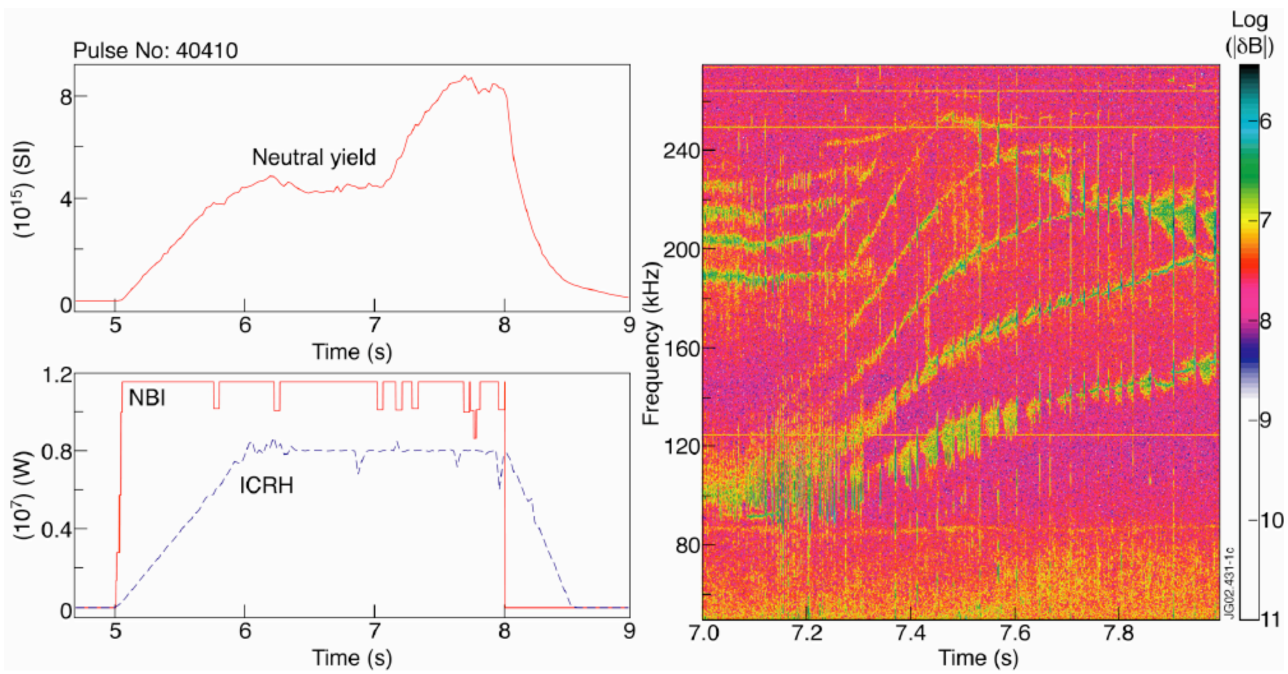



*N. Gorelenkov*

- **ITER will have 1 MeV negative-ion neutral beams for current drive & heating**
  - Theory predicts that these NNBI energetic ions can drive TAE instability, comparable to alpha particles
  - With the beam ion drive included, the stability prediction for ITER changes (at 20 keV) from marginality to definite instability
  - A model quasi-linear calculation predicts negligible to modest losses

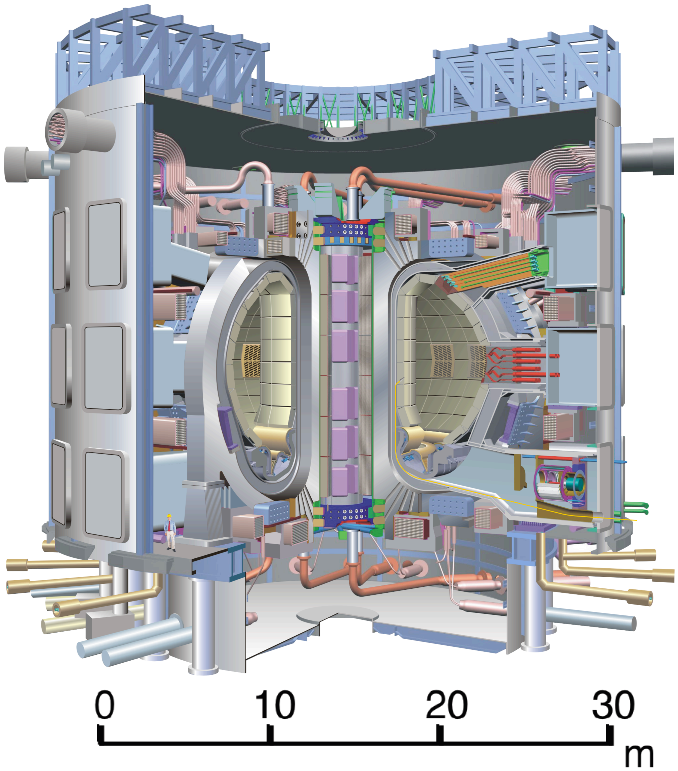
# Alfvén instabilities as plasma diagnostic

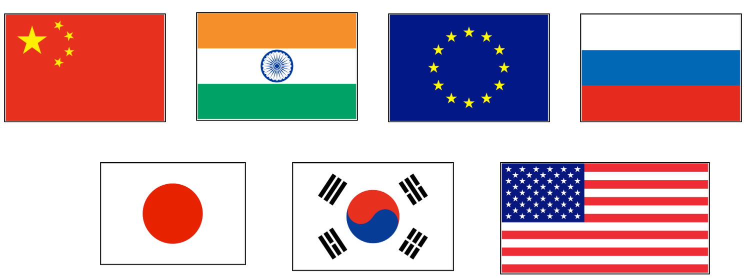
- **Internal transport barrier (ITB) triggering event**
  - "Grand Cascade" (many simultaneous n-modes) occurrence is coincident with ITB formation (when $q_{min}$ passes through integer value)
  - Being used on JET as an internal diagnostic to monitor $q_{min}$
  - Can  create ITB by application of main heating shortly before a Grand Cascade is known to occur



# TAE RESEARCH PROGRAM

# ITER will demonstrate the scientific and technological basis for fusion energy



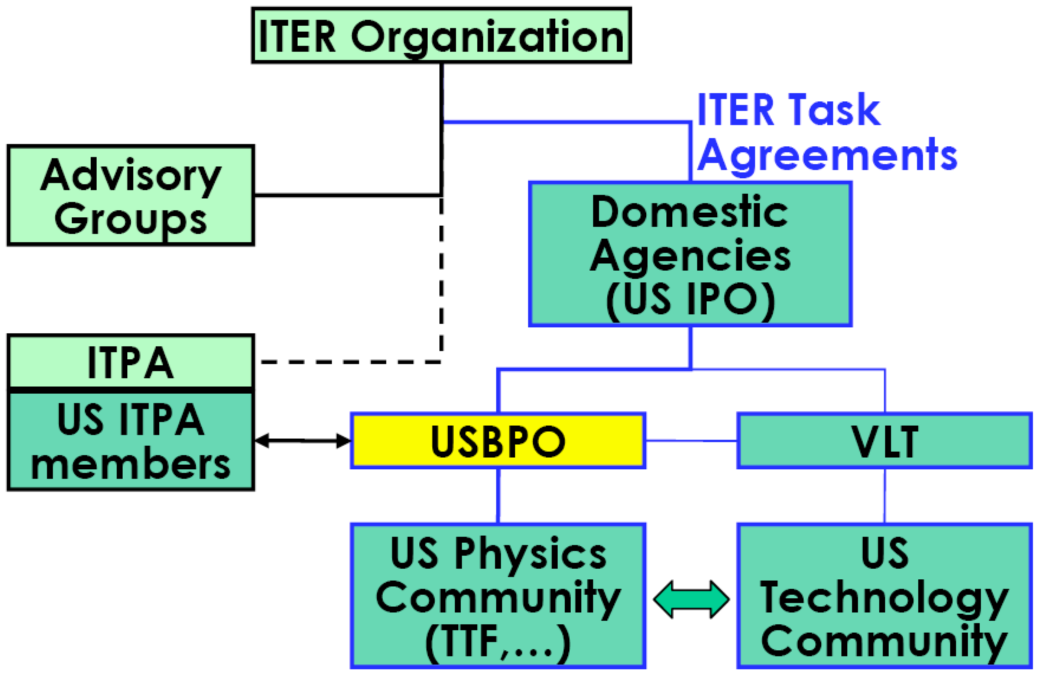
- **ITER ("the way") is essential next step in development of fusion**
  - Today: 10 MW, 1 sec, gain = 1
  - ITER: 500 MW, > 400 sec, gain ≥ 10
- **The world's biggest fusion energy research project ("burning plasma")**
  - 15 MA plasma current, 5.3 T magnetic field, 6.2 m major radius, 2.0 plasma minor radius, 840 $m^3$ plasma volume, superconducting
  - €10B to construct, then operate for 20 years (First Plasma" in 2019, DT in 2027)
- **An international collaboration**
  - 7 partners, 50% of world's population
  - EU the host Member, sited in France
  - Excellent example of US involvement in big-science international physics collaboration (cf. Large Hadron Collider, ALMA telescope)

# USBPO: physics support for ITER

- **U.S. Burning Plasma Organization is community-based**
  - Mission: *Advance the scientific understanding of burning plasmas and ensure the greatest benefit from burning plasma experiments by coordinating relevant U.S. fusion research with broad community participation*
- **Broad community participation:**
  - Regular members (316 from 55 institutions)
  - Associate members (15 from 9 non-US institutions)
- **USBPO web site (www.burningplasma.org)**
  - All presentations, white papers, progress reports are publicly available
  - *eNews* monthly newsletter: 480 subscribers (from 95 institutions)
    - "Director's Corner" column, feature articles, ITPA meeting reports, calendar of fusion events, research highlights, community reports

# USBPO role in ITER support

- **US ITER Project Office (ORNL)**
  - Main link to ITER
  - Provides hardware & technical contributions
- **USBPO**
  - Coordinates US burning plasma physics research
  - USBPO director is also the US ITER Project Office chief scientist
  - Companion to Virtual Laboratory for Technology



# Expertise of USBPO topical groups

USBPO U.S. BURNING PLASMA ORGANIZATION

Jim Van Dam (Director)
Chuck Greenfield (Deputy Director)
Nermin Uckan (Assistant Director for ITER Liaison)

Council:
Mike Mauel (Chair)
Michael Bell (Vice Chair)
+10 members at large

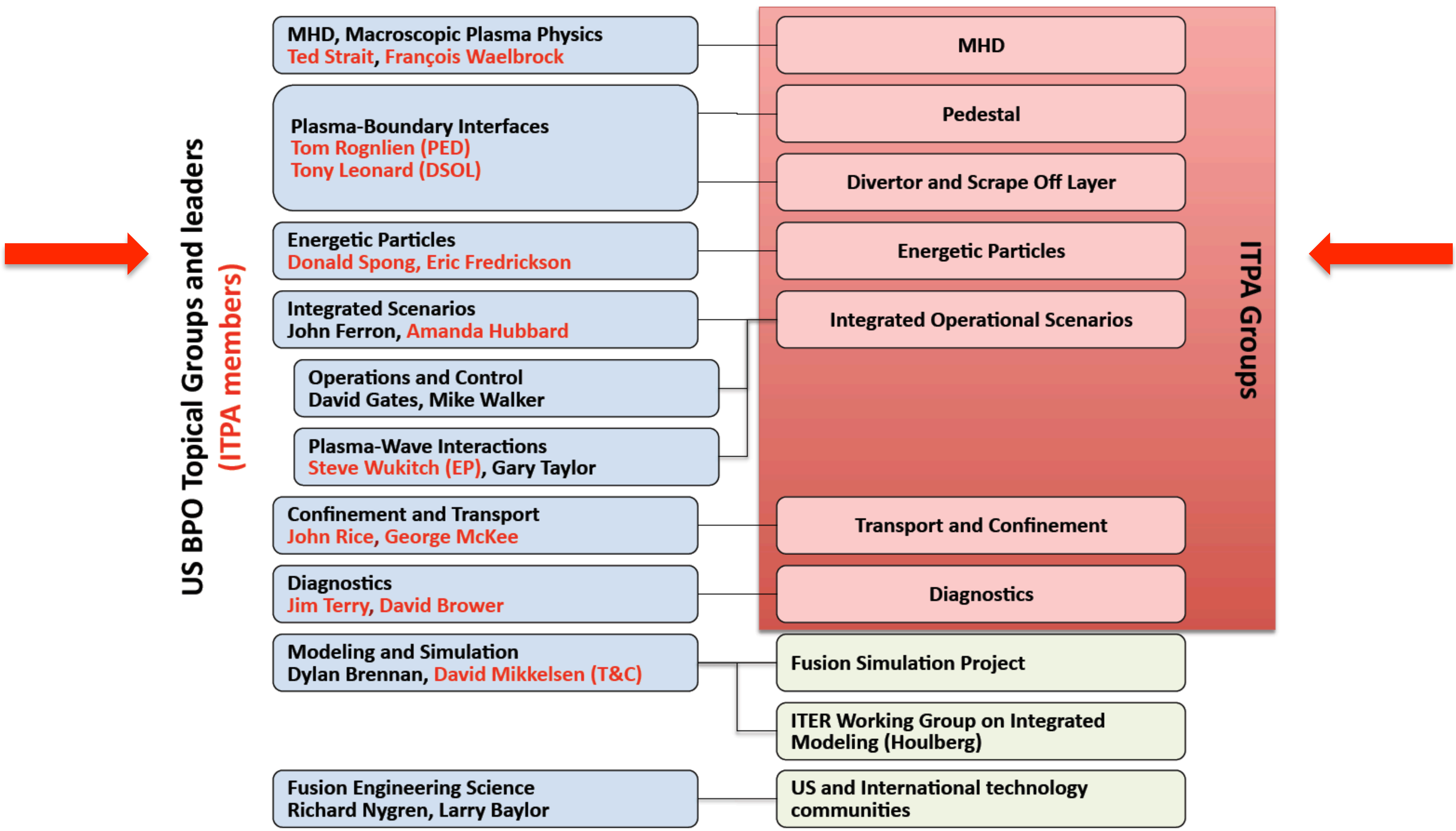
Executive Committee members in red

## Research Committee made up of Topical Group Leadership

| | |
|---|---|
| MHD, Macroscopic Plasma Physics<br>Ted Strait, François Waelbrock | Integrated Scenarios<br>John Ferron, Amanda Hubbard |
| Boundary<br>Tom Rognlien, Tony Leonard | Operations and Control<br>David Gates, Mike Walker |
| Fusion Engineering Science<br>Richard Nygren, Larry Baylor | Modeling and Simulation<br>Dylan Brennan, Dave Mikkelsen |
| Diagnostics<br>Jim Terry, David Brower | Confinement and Transport<br>John Rice, George McKee |
| Plasma-Wave Interactions<br>Steve Wukitch, Gary Taylor | Energetic Particles<br>Donald Spong, Eric Fredrickson |

ITPA

# ITPA also has Energetic Particles group




Mar 2010: Plasma-Boundary Interfaces topical group was renamed “Pedestal and Divertor/SOL”

# 2010 IISS

- **4th ITER International Summer School held in US last year**
  - May 31-June 4, University of Texas
  - Sponsors: National Instruments, USBPO, French Embassy in US, ....
- **Theme:** ***MHD and Plasma Control in Magnetic Fusion Devices***
- **Participation**
  - 133 participants from 17 countries and 48 institutions

*"Fusion is the future,*
*and the future is in your hands."*

20 lecturers from 7 countries & ITER

4 computer lab sessions

# Burning plasma at APS-DPP Meeting

- **Town Meeting on ITER Status (Tues, Nov 9, 2010)**
  - Gyung-Su Lee (MAC): *New ITER Baseline and Risk Assessment*
  - Alberto Loarte (ITER): *Scientific Status of ITER*
  - Brad Nelson (USIPO): *US Engineering and Technology R&D for ITER*
  - Jim Van Dam (USBPO): *US Scientific Contributions to ITER R&D*
  - Discussion session: moderator Mike Mauel (USBPO Council)

- **Two contributed ITER oral sessions (@ 11 ten-minute talks)**

- **Town Meeting talks are posted on USBPO web site**

- **Likewise, being organized for 2011 APS-DPP Meeting**

# SUMMARY

- **Burning plasma studies on ITER open up new regime of plasma physics of an exothermic medium**
  - A "grand challenge" problem, with potential social benefit
- **Dramatic scientific progress in last two decades has laid the foundation for burning plasma experiments**
  - Coordinated efforts of Experiments, Diagnostics, Theory, and Simulations to create validated predictive models of plasma behavior
  - Alfvén wave instabilities are important topic for burning plasmas
- **Construction has begun of long-awaited world's first burning plasma experiment: ITER**
  - Many exciting near/longer-term research issues in burning plasma science for ITER operation and next-generation experiments (DEMO)

# References: Burning Plasmas

- *Final Report–Workshop on Burning Plasma Science: Exploring the Fusion Science Frontier* (2000) http://fire.pppl.gov/ufa_bp_wkshp.html
- *Review of Burning Plasma Physics* (Fusion Energy Sciences Advisory Committee, 2001) http://fire.ofes.fusion.doe.gov/More_html/FESAC/Austinfinalfull.pdf
- *Burning Plasma: Bringing a Star to Earth* (National Academy of Science, 2004)
- R. Hawryluk, Results from Deuterium-Tritium Tokamak Confinement Experiments, Reviews of Modern Physics v. 70, p. 537 (1998)
- Presentations at USBPO Burning Plasma Workshop 2005 www.burningplasma.org/reference.html (energetic particle physics plenary talk, break-out group presentations, and summary)
- ITER Physics Basis, Chap. 5 (Energetic Particles), Nuclear Fusion (1999); *Progress in the ITER Physics Basis*, Nuclear Fusion (2007)
- R. J. Fonck, Scientific Developments in the Journey to a Burning Plasma, invited talk at 2009 APS Spring Meeting (http://burningplasma.org/reference.html)

# Some classic TAE references

- **Fast ion excitation of Kinetic Alfvén Wave**
  - M. N. Rosenbluth and P. H. Rutherford, "Excitation of Alfvén Waves by High-Energy Ions in a Tokamak," Phys. Rev. Lett. **34**, 1428 (1975)
- **Existence of discrete TAE mode**
  - C. Z. Cheng, L. Chen, and M. S. Chance, "High-n Ideal and Resistive Shear Alfvén Waves in Tokamaks," Ann. Phys. (NY) **161**, 21 (1985)
- **TAE excitation by alpha particles**
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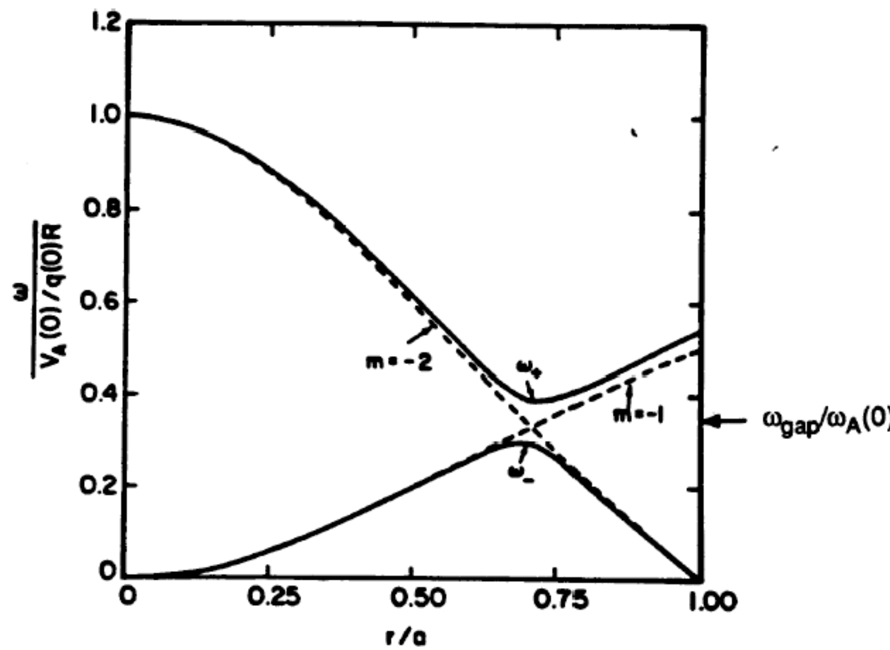
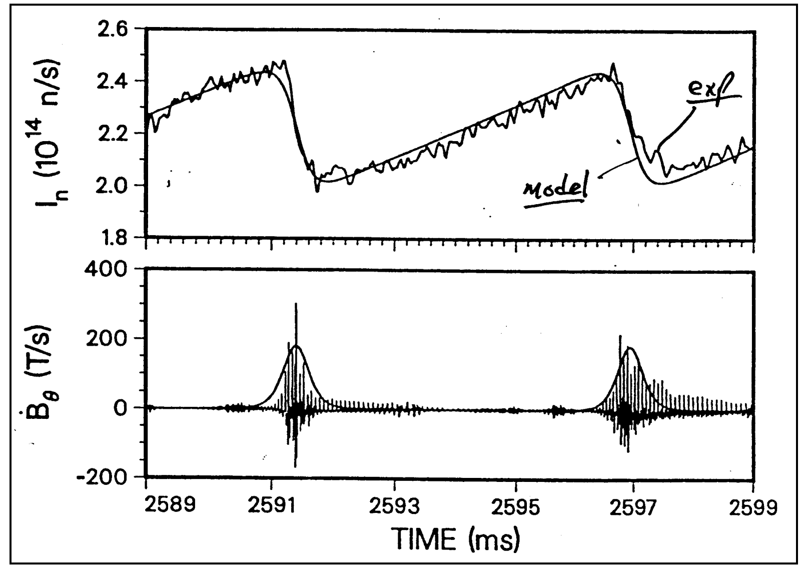
# Exercise #1: TAE frequency and gap

- For the Toroidal Alfvén Eigenmode, calculate the q-value where the gap is located and estimate the typical TAE mode frequency.
  - Repeat this exercise for the Ellipticity- and Triangularity-induced Alfvén Eigenmodes



# Exercise #2: Nonlinear fishbone cycle

- Consider a simple model for the fishbone cycle [R. White]. Assume the trapped fast ions are deposited at rate S until the threshold beta $\beta_c$ is reached. Model the losses as a rigid displacement of the trapped particles toward the wall. Equations for the trapped particle beta $\beta$ and the mode amplitude A are:




$$\frac{d\beta}{dt} = S - A\beta_c \ , \quad \frac{dA}{dt} = \gamma_0\left(\frac{\beta}{\beta_c} - 1\right)A$$

  - Show that the solution of these equations will be cyclic. [Hint: Approximately plot $\beta$ and A as functions of time. Then, from the equations, construct a function $F(\beta, A)$ that satisfies $\partial F/\partial t = 0$. Approximately plot the contours F = constant in $\beta$-A phase space. Show that F is minimum when $\beta = \beta_c$ and $A=S/\beta_c$.]

- If the losses are diffusive, rather than rigid, then $d\beta/dt = S - A\beta$. In this case, show that $\partial F/\partial t \leq 0$, with $\partial F/\partial t=0$ only at $\beta=\beta_c$, and that the solution spirals toward this fixed point.